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**Term Structure Rules
for Monetary Policy**

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TERM STRUCTURE RULES FOR MONETARY POLICY

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Abstract

This paper studies two types of interest rate rules that involve long-term nominal interest rates in the context of a New Keynesian model. The first type considers the possibility of adding longer-term rates to the list of variables the central bank reacts to in setting its short-term rate. The second type considers Taylor-type rules that are expressed in terms of interest rates of different maturities, which are operationally equivalent to more complex rules expressed in terms of the short-term rate. It is shown that both types of rules can give rise to a unique rational expectations equilibrium in large regions of the policy-parameter space. The normative evaluation shows that under certain preferences of the monetary authority, policy rules of the second type produce better results than the standard Taylor-type rule.

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TERM STRUCTURE RULES FOR MONETARY POLICY

Mariano Kulish

1. Introduction

The transmission mechanism of monetary policy is traditionally perceived as going from a short-term nominal interest rate to a long-term real interest rate that influences aggregate demand. Recently, there have been proposals involving the use of nominal long-term interest rates for the conduct of monetary policy. On the one hand, as Goodfriend (1993) notes, long-term nominal interest rates may contain information about long-term inflationary expectations, thus making them useful indicators for the central bank. On the other hand, the potential of long-term rates to directly influence economic activity motivates the study of policy rules which incorporate longer-term rates.¹ The goal of this paper is to study monetary policy rules that involve long-term nominal interest rates in these two distinct roles.

1.1 Inflationary Expectations: Type-1 Rules

The Fisher decomposition reveals that two terms are crucial for the equilibrium determination of nominal interest rates: an expected real rate and an expected inflation term. Thus, policy-makers might want to use long-term nominal interest rates to help measure the private sector's long-term inflationary expectations. To the extent that the predominant force moving long-term yields is the expected inflation component, a monetary authority interested in keeping inflation under control might be interested in the use of reaction functions that incorporate long-term yields as arguments. Interestingly, McCallum (1994) has shown that a monetary policy rule that responds to the prevailing level of the spread between a long-term rate and a short-term rate can rationalise an important empirical failure

¹ See Bernanke (2002) and Clouse *et al* (2003).

of the expectations hypothesis.² However, such behaviour by the central bank raises two important considerations.

First, one can show that the theory of the term structure that emerges from optimising behaviour in a New Keynesian model is the expectations hypothesis. Thus, the market determines nominal long-term interest rates as the average expected level of nominal short-term interest rates over the maturity horizon under consideration. A monetary policy reaction function that includes a long-term rate immediately raises the question of whether or not a unique rational expectations equilibrium (REE) exists in this case. The question is important since the combined power of the expectations hypothesis and the proposed monetary policy rule might give rise to self-fulfilling prophecies in the equilibrium determination of the yield curve.³ What are the conditions that guarantee uniqueness of the REE when the central bank's actions depend on the level of a long-term interest rate in addition to inflation and the output gap?

Second, assuming that the conditions that ensure a unique REE exist, is it desirable to have the central bank responding to long-term rates in this way? And if so, which is the best maturity length for the monetary authority to react to?

To study these questions I propose a modification of a standard Taylor rule that adds a long-term rate as an additional variable to which the central bank adjusts its short-term rate; hereafter, these are referred to as type-1 rules. In the context of a standard New Keynesian model, I show that there are large and empirically plausible regions of the policy-parameter space where a unique REE exists when the central bank conducts policy in this manner. In addition, I find that reacting to movements in long-term rates does not improve the performance of the central

² This failure is related to the magnitude of the slope coefficients in regressions of the short rate on long-short spreads. A partial equilibrium interpretation of the expectations hypothesis implies that the slope coefficient, b , in a regression of the form, $\frac{1}{2}(R_{1,t} - R_{1,t-1}) = a + b(R_{i,t} - R_{1,t-1}) + shock$, should have a probability limit of 1. Many empirical findings in the literature yield a value for b considerably below 1. As shown by McCallum (1994) the expectations hypothesis is consistent with these findings if it is recognised that the term premium follows an exogenous random process and monetary policy involves smoothing of the instrument as well as a response to the level of the spread.

³ In fact, Bernanke and Woodford (1997) show that a policy rule, in which the short-term rate reacts only to a long-term rate, is unable to yield a unique REE. However, in this paper I consider more general rules that nest Bernanke and Woodford's case as a special one.

bank relative to the standard Taylor rule, regardless of the maturity length in question.

1.2 Long-term Interest Rates: Type-2 Rules

It has been suggested that long-term rates might be used as instruments of monetary policy in Taylor rules (that is, where the long rate replaces the short rate as the argument of the rule; hereafter referred to as type-2 rules). Various aspects of this proposal have been studied by Kulish (2005) and McGough, Rudebusch and Williams (2005). This does not imply, however, that type-2 rules require monetary authorities to alter their current operating procedures – that is, by switching to a longer-term nominal interest rate as their instrument. Indeed, interest rates of various maturities are linked by the expectations hypothesis in the New Keynesian model so that long-term interest rate rules could alternatively be written as more complicated short-term rate rules.⁴

In this paper I study the determinacy properties of the REE as well as the performance of long-term interest rate rules. This study is interesting in its own right, but it is also of general theoretical importance to monetary economics for the following reason.

One might initially suspect that a unique REE will not arise if the monetary authority decides to use a two-period interest rate rule. The reason is that, in a context in which the expectations hypothesis holds true, there will exist infinite paths for the one-period rate that satisfy the central bank's setting of the two-period rate. Notice that abstracting from a term premium and default risk, the two-period rate is an average of the current one-period rate and the current expectation of the one-period rate in the next period. In other words, if the central bank wishes to set the two-period interest rate at, say, 5 per cent, then in equilibrium the one-period rate could follow any number of paths provided that the average for the one-period rate's path is 5 per cent.

This argument suggests that a unique equilibrium would not exist when the central bank uses a rule for the two-period rate, or its equivalent in terms of the

⁴ In other words, the choice of operating instrument when constrained to a functional form of the policy rule is equivalent to some choice of functional form when constrained to one particular instrument.

one-period rate. Imagine for a moment that this is indeed the case and recall that almost all of the modern dynamic discrete time models of monetary economics are based on a quarterly frequency. So in theory the operating instrument is usually a 3-month interest rate, whereas the actual operating instrument of monetary policy in most developed economies is an overnight rate. The inability to map the theoretical operating instrument with the actual one would be a damning result. Fortunately, this suspicion turns out to be incorrect. In fact, as shown below, large and empirically plausible regions of the policy-parameter space for long-term interest rate rules yield a unique REE for the economy.⁵ Thus, the results of this paper provide a theoretical foundation for the study of monetary models at different frequencies.

These alternative monetary policy rules are studied in the context of a New Keynesian model for the following reasons. First, a standard version of the New Keynesian model embodies the traditional view of the monetary transmission mechanism, in which the central bank controls the short-term nominal interest rate, while the long-term real interest rate determines aggregate demand. Second, as emphasised by Goodfriend and King (1997), the New Keynesian model has achieved a certain consensus in the macroeconomic literature, to the point that the authors refer to it as the New Neoclassical Synthesis. Third, the New Keynesian model is now extensively used for theoretical analysis of monetary policy.⁶

The rest of the paper is organised as follows. Section 2 describes the model. Section 3 discusses determinacy of the REE under term structure rules and their implications for the dynamic behaviour of the economy.⁷ Section 4 studies the performance of these alternative term structure rules against two benchmarks: the robust optimal policy rule and the standard Taylor-type rule. Section 5 summarises the main results.

⁵ This result may also be of practical importance in light of the zero-bound/liquidity trap problem. See Bernanke (2002), Kuliš (2005), and McGough *et al* (2005) for more details on this issue.

⁶ See Clarida, Galí and Gertler (1999), Goodfriend and King (1997), Walsh (1998), Woodford (2003) and the references therein.

⁷ Elsewhere in the literature, Gallmeyer, Hollifield and Zin (2005) have proposed the term ‘McCallum rules’ to refer to a monetary policy rule in which the instrument is sensitive to the slope of the yield curve. Here, ‘term structure rules’ refer to monetary policy rules that involve long-term rates in a more general way. In my terminology, a McCallum rule is a special case of a term structure rule.

2. The New Keynesian Model

The model presented here is a standard New Keynesian model with an extended set of equilibrium conditions in order to allow for an explicit consideration of the term structure of interest rates. Instead of working through the details of the derivation, I present the key aggregate log-linear relationships.⁸

The aggregate demand schedule implies that the current level of the output gap, x_t , depends on the expected future level of the output gap and the one-period real interest rate:

$$x_t = -(R_{1,t} - E_t \pi_{t+1}) + E_t x_{t+1} + \mu_g (1 - \phi) g_t - (1 - \rho) a_t \quad (1)$$

where $R_{1,t}$ is the one-period nominal interest rate; π_t is the inflation rate during period t ; a_t is a technology shock with persistence governed by ρ ; and g_t is a preference shock with persistence governed by ϕ and size μ_g .

It can be shown that the theory of the term structure of interest rates that emerges from optimising behaviour in the context of this model is the expectations hypothesis. The nominal interest rate at t associated with a zero-coupon bond that promises to pay one dollar at the end of period $t + i - 1$ is given by

$$R_{i,t} = \frac{1}{i} E_t \sum_{k=1}^i R_{1,t+k-1}, \forall i \geq 2 \quad (2)$$

Firms are assumed to operate in an environment characterised by monopolistic competition in the goods market and by price stickiness. Factor markets are assumed to be competitive and goods are produced with a constant returns-to-scale technology. Following Calvo (1983), it can then be shown that the above assumptions produce the log-linear New Phillips curve given by

$$\pi_t = \lambda x_t + \beta E_t \pi_{t+1} + \mu_v v_t \quad (3)$$

where v_t is a cost-push shock with size μ_v . The parameter $\lambda > 0$ governs how inflation reacts to movements of output from its natural level. A larger value of λ implies that there is a greater effect of output on inflation. In this sense, prices

⁸ See Clarida *et al* (1999), Ireland (2004), and Woodford (2003) for a more detailed discussion of the New Keynesian model.

may be viewed as adjusting faster. The household's discount factor β is restricted to lie between 0 and 1.

Finally, to close the model we need assumptions about the behaviour of the monetary authority. The standard case characterises monetary policy as a commitment to the following Taylor-type rule:

$$R_{1,t} = \tau R_{1,t-1} + \alpha \pi_t + \delta x_t + \mu_b b_t \quad (4)$$

where b_t is a monetary policy shock whose size is governed by μ_b .

In the case in which the monetary authority adds a long-term rate as an additional variable to which it reacts, the policy rule can be characterised by

$$R_{1,t} = \tau R_{1,t-1} + \alpha \pi_t + \delta x_t + \gamma R_{i,t} + \mu_b b_t \quad (5)$$

Throughout the paper, policy rules that allow a reaction to long-term rates are labeled type-1 rules. I study these rules of type-1 for maturities 2, 4, 12, 20 and 40 which, for a quarterly frequency, correspond to a term structure composed of bonds with maturities of 6 months, 1 year, 3 years, 5 years and 10 years, respectively.

When the central bank replaces the short-term rate in the standard Taylor rule with a long-term rate, monetary policy follows a rule of the form

$$R_{i,t} = \tau R_{i,t-1} + \alpha \pi_t + \delta x_t + \mu_b b_t \quad (6)$$

Policy rules like this are labeled type-2 rules. As before, the selected term structure for type-2 rules is 2, 4, 12, 20 and 40.

Notice that as interest rates of various maturities are linked by the expectations hypothesis, whatever outcome a type-1 or type-2 rule produces, it could alternatively be obtained using some given rule for the short-term rate. For example, the central bank could achieve the same equilibrium allocation either by using a type-2 rule for $R_{2,t}$, or by using a rule for the short-term of the form

$$R_{1,t} = \tau R_{1,t-1} + \tau E_{t-1} R_{1,t} - E_t R_{1,t+1} + 2\alpha \pi_t + 2\delta x_t + 2\mu_b b_t \quad (7)$$

Hence, one could view the exercise either as an analysis of different policy rules for the short rate, or as a comparison of Taylor-type rules involving various longer-term rates.

Finally, the stochastic block of the model is assumed to behave as given by

$$\begin{aligned}
 a_t &= \rho a_{t-1} + \varepsilon_t^a \\
 b_t &= \varepsilon_t^b \\
 g_t &= \phi g_{t-1} + \varepsilon_t^g \\
 v_t &= \theta v_{t-1} + \varepsilon_t^v
 \end{aligned} \tag{8}$$

where the parameters are restricted as follows: $|\rho| < 1$, $|\phi| < 1$, $|\theta| < 1$, and the independently and identically distributed shocks ε_t^a , ε_t^b , ε_t^g and ε_t^v have normal distributions, zero mean, and standard deviations given by, σ_{ε^a} , σ_{ε^b} , σ_{ε^g} , and σ_{ε^v} , respectively.

The model is calibrated to roughly match key features of the Australian and the US economies. The parameters β and λ are fixed throughout the study. These parameters are set to 0.99 and 0.14, respectively, as usually done in the literature. The value for λ implies an expected price-contract length of one year. The shocks associated with the parameters ρ , ϕ and θ are thought to be highly persistent innovations.⁹ For this reason they are set to 0.95. Finally, the parameters that control the size or standard deviation of the remaining shocks are calibrated as follows. The standard deviation of the technology shock, σ_{ε^a} , is set to 0.7 following Cooley and Prescott (1995). Then, in the case of a Taylor rule with $\tau = 0.5$, $\alpha = 0.6$ and $\delta = 0.0009$, the values of μ_b , μ_g and μ_v are chosen so as to approximate the volatility of the output gap, the interest rate and inflation in the data.¹⁰

For Australia, the output gap is constructed as the log difference between the quarterly real non-farm output and real potential quarterly output as calculated by the model of Stone, Wheatley and Wilkinson (2005). Inflation is measured as the quarterly change of the weighted-median CPI, and the interest rate is taken to be the nominal 90-day bank bill rate. For the US, the output gap is constructed as the log difference between the seasonally adjusted quarterly real GDP and real potential quarterly GDP taken from the Congressional Budget Office. Inflation

⁹ See Ireland (2004) and Cooley and Prescott (1995).

¹⁰ σ_{ε^b} , σ_{ε^g} , and σ_{ε^v} are set to 1.

is measured as the quarterly change of the GDP implicit price deflator and the interest rate is the 3-month Fama and Bliss zero-coupon bond yield data from the Center for Research in Security Prices.

3. Equilibrium Determinacy

Under appropriate identifications, the model can be written in matrix form as

$$E_t \mathbf{s}_{t+1} = \mathbf{K} \mathbf{s}_t + \mathbf{L} \mathbf{v}_t \quad (9)$$

where: $\mathbf{s}_t = (\mathbf{z}'_t \ \mathbf{p}'_t)'$; \mathbf{z}_t is a $(m \times 1)$ vector of pre-determined variables at t ; \mathbf{p} is a $(n \times 1)$ vector of non-pre-determined variables at t ; and \mathbf{v}_t is a $(k \times 1)$ vector of exogenous variables.¹¹

Let \bar{n} be the number of eigenvalues of \mathbf{K} outside the unit circle. There are three cases to consider. If $\bar{n} = n$ then there is a unique equilibrium solution. If $\bar{n} > n$ then an equilibrium solution does not exist, and if $\bar{n} < n$ then there is an infinite number of equilibrium solutions.¹²

As the reader might appreciate, the study of uniqueness becomes analytically intractable, especially as we move towards larger maturities. For this reason, I resort to a numerical study of the problem. Nevertheless, as shown below, interesting numerical patterns emerge from this study.

3.1 Type-1 Rules: Reacting to Long-term Interest Rates

In this sub-section I study the conditions that support a unique REE for the class of type-1 rules given by $R_{1,t} = \tau R_{1,t-1} + \alpha \pi_t + \delta x_t + \gamma R_{i,t}$. Figure 1 shows the regions of uniqueness for given values of τ and δ in the space of α and γ .¹³ A number of interesting features of this type of policy rule are worth highlighting.

¹¹ When monetary policy uses a Taylor rule that involves interest rate smoothing, then the interest rate is a pre-determined variable, inflation and output are jump (or non-pre-determined) variables, and shocks are the exogenous variables.

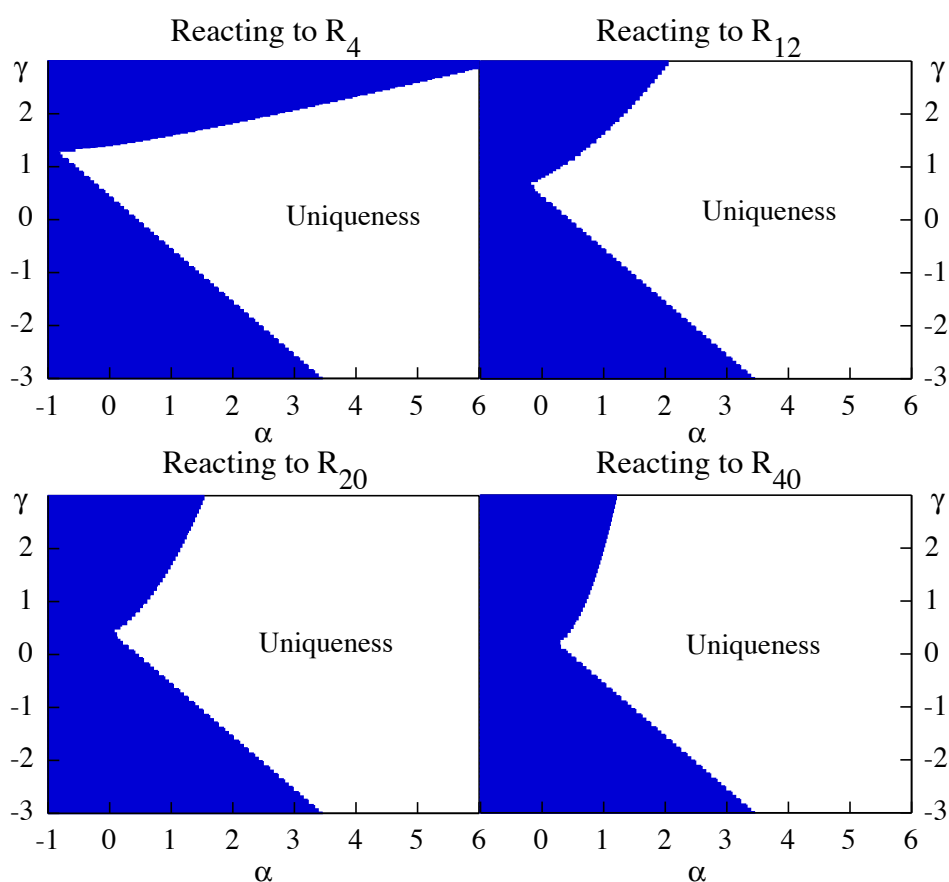
¹² See Blanchard and Kahn (1980) for a detailed presentation.

¹³ Figure 1 shows regions of uniqueness for R_4 , R_{12} , R_{20} , and R_{40} . Kulish (2005) contains figures for R_2 as well.

Notice that the critical contour, for which the crucial eigenvalue of the \mathbf{K} matrix is 1, has a downward- and an upward-sloping part in each case. The downward-sloping portion of the contour reveals that there is a trade-off, in terms of assuring a unique equilibrium, between the reaction to current inflation, α , and the reaction to the long-term nominal interest rate, γ .

Figure 1: Regions of Uniqueness for Type-1 Rules

$$\delta = \tau = 0.5$$



Further numerical exploration shows that in the downward-sloping part of the contour, a condition of the form $(\tau + \alpha + \gamma > 1)$ is necessary for determinacy, regardless of the maturity length in question. Note that the Taylor principle $(\alpha + \tau > 1)$, that the short-term rate must rise sufficiently in the long run in response to movements in inflation so as to increase real rates, no longer holds in this case. The upward-sloping section of the contour shows that for a given α , as γ becomes ‘too large’, the policy rule is unable to produce a unique outcome.

The intuition behind this result is the following. For simplicity, take the two-period interest rate and use Equations (1) and (2) to write

$$R_{2,t} = \frac{1}{2} E_t (x_{t+2} - x_t + \sum_{k=1}^2 \pi_{t+k})$$

The above expression shows that by reacting to the two-period interest rate, the monetary authority is implicitly reacting to the average expected path of inflation in the following two periods. This explains why the Taylor principle is modified to a more general condition of the form $(\alpha + \tau + \gamma > 1)$ in the downward-sloping part of the contour. In this sense there is a trade-off for assuring a unique equilibrium between the reaction to the long-term rate and current inflation.

To gain intuition about the upward-sloping portion of the contour, recall that, according to the expectations hypothesis, we may alternatively express $R_{2,t}$ as

$$R_{2,t} = \frac{1}{2} (R_{1,t} + E_t R_{1,t+1})$$

The problem is that a ‘too large’ value of γ allows self-fulfilling expectations to take place. To see why, observe that expectations that interest rates will be high become self-fulfilling, because the expectations of high short-term rates in the future causes long-term rates to rise, leading the monetary authority to raise short-term rates. Thus, in this case the monetary authority validates the initial expectation that short-term rates will be high. The upward-sloping part of the contour shows that there is a complementarity between α and γ . In this region, a higher value of γ requires a stronger response of the short-term rate to inflation in order to avoid the possibility of self-fulfilling expectations. It is in this respect that the Taylor principle breaks down for type-1 rules.

Further numerical exploration shows that in the upward-sloping part of the contour, higher values of τ and δ permit, for a given value of α , a higher value of γ . Notice that the Fisher decomposition implies, without loss of generality, that reacting positively to R_2 is equivalent to an implicit negative reaction to the current output gap. To see this, consider Equation (5), the type-1 policy rule with i equal to 2, and rewrite it with the help of the Fisher decomposition as

$$R_{1,t} = \tau R_{1,t-1} + \alpha \pi_t + \frac{\gamma}{2} \sum_{k=1}^2 E_t \pi_{t+k} + (\delta - \frac{\gamma}{2}) x_t + \frac{\gamma}{2} E_t x_{t+2}$$

For this reason, higher values of δ allow for higher values of γ without implying a negative reaction to the current output gap.¹⁴

Figure 1 reveals that as we move towards policy rules that involve larger maturity rates, the upward-sloping region of the critical contour increases its slope. So, for a given value of α it is possible to ensure unique solutions with even higher values of γ as the maturity lengthens. The fact that the interest rates in the policy rule are further apart in terms of maturity explains this result. When monetary policy reacts to very long-term rates, expectations can become self-fulfilling if the reaction of the short-term rate to this movement is sufficiently strong so as to feed through the term structure with enough strength to move the very long-term rate in a self-validating manner. So, as a stylised numerical observation, if the condition $\tau + \alpha + \gamma > 1$ is satisfied (for given values of the other parameters) the larger the maturity of the long-term rate in a type-1 rule, the larger the value of γ that supports a unique solution of the system.

Up to this point I have shown that there are large and empirically plausible regions of the parameter space for which type-1 rules yield a unique REE. This is important because rules that support multiple solutions are problematic. The mere fact that such a rule may be consistent with a potentially desirable equilibrium is of little importance if it is also equally consistent with other, much less desirable equilibria. A rule that implies indeterminacy is consistent with a large set of equilibria, including ones in which the fluctuations of endogenous variables are arbitrarily large relative to the size of fluctuations in the exogenous shocks.¹⁵ In general, variables for which there may be arbitrarily large fluctuations due to self-fulfilling expectations include those that enter the loss function of the monetary authority. Hence, at least some of the equilibria consistent with the rule are likely to be less desirable, in terms of the loss function, than the unique equilibrium associated with a rule that guarantees a unique solution.

¹⁴ It can be shown in the context of the standard case (that is, with a policy rule of the form $R_{1,t} = \tau R_{1,t-1} + \alpha \pi_t + \delta x_t$) that under the proposed calibration of the model (in particular, $\lambda = 0.14$ and $\alpha > 1$), there are multiple equilibria so long as $\delta < 0$.

¹⁵ See Bernanke and Woodford (1997) for a formal description of a ‘sunspot’ equilibrium. See also Woodford (2003, chapter 4).

For these reasons, the normative analysis restricts its attention to rules that imply a unique equilibrium. The question of the benefits of type-1 rules that yield a unique solution is taken up in Section 4.

3.2 Type-2 Rules: Long-term Interest Rate Rules

Here I study the conditions under which a unique REE exists for the class of type-2 rules given by $R_{i,t} = \tau R_{i,t-1} + \alpha \pi_t + \delta x_t$. In this case, a longer-term interest rate enters in the left-hand side of the reaction function used by the monetary authority. Recall that this does not imply that the monetary authority needs to change its operating procedures in any respect. The central bank can still use the short-term rate as its operating instrument to achieve the same equilibrium allocation. Rather, this is merely a more straightforward way to consider a rule which, when expressed in terms of the response of the short rate, can be quite complex.

Figure 2 shows the regions in the space of α and δ in which a unique REE exists for $\tau = 1/2$. As can be observed, significantly large regions of the parameter space exist that produce a unique REE.¹⁶ Notice that the regions of determinacy in the positive quadrants remain unchanged whatever the term of the interest rate chosen for the rule. Interestingly, Figure 2 also shows that, in the positive quadrants, the Taylor principle holds for the selected value of τ . Notice that $\alpha + \tau > 1$ is required for determinacy in these cases. In other words, for all rules and $\tau = 1/2$, determinacy of the REE requires $\alpha > 1/2$. Although the numerical exercise so far suggests that the Taylor principle generalises to longer maturities, this is true only in some regions of the policy-parameter space.

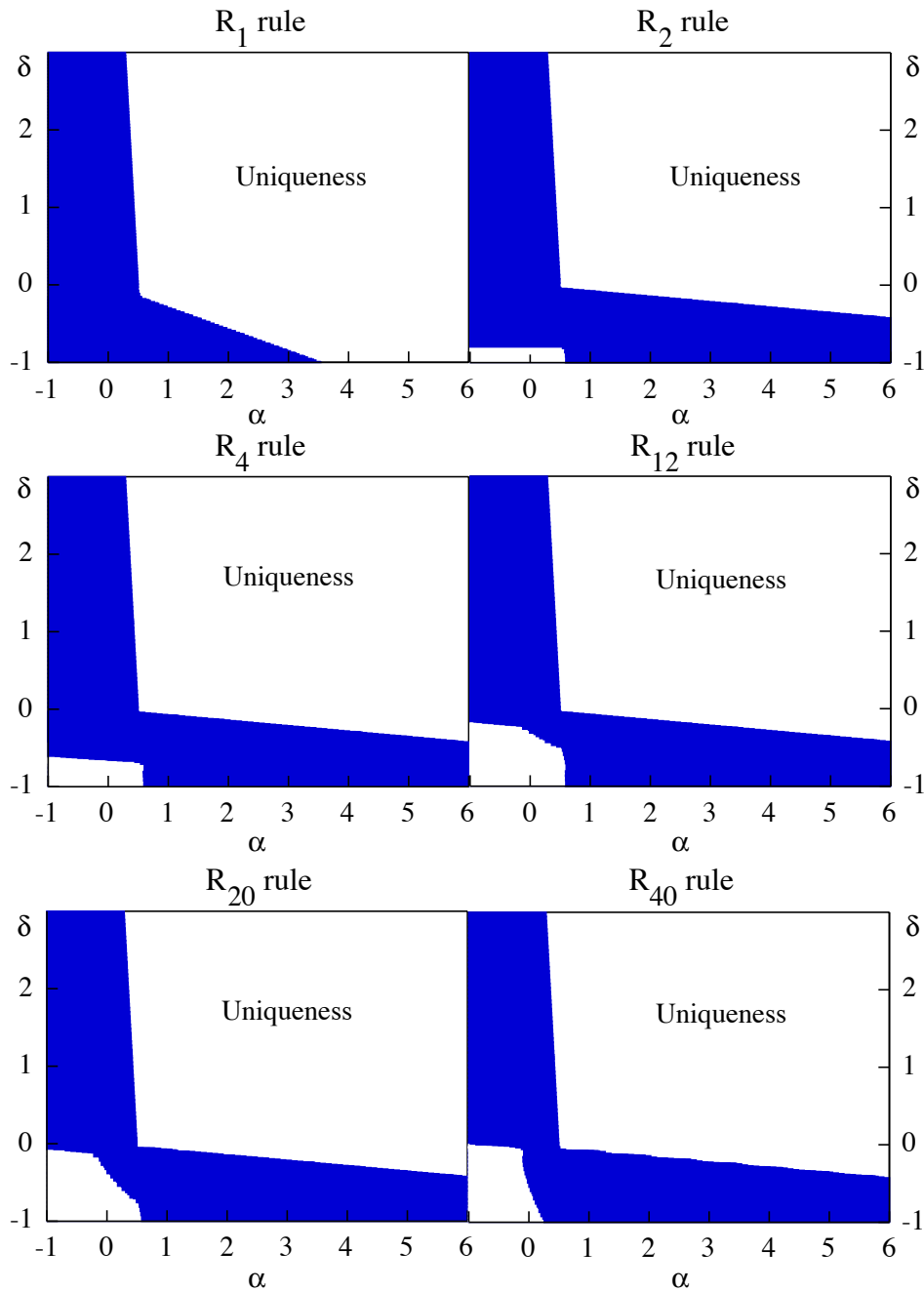
Further exploration in the space of α and τ shows that the Taylor principle eventually breaks down for longer-term interest rates rules by displaying an upward-sloping section of the critical contour. This complementarity between α and τ shows up for sufficiently positive values of δ . Interestingly, in the standard Taylor rule case, the critical contour's slope is always -1 with its equation given by $\tau = 1 - \alpha$.¹⁷

¹⁶ McGough *et al* (2005) also found regions of uniqueness for similar rules in an environment of the same kind.

¹⁷ An earlier version of Kulish (2005) contains figures in $\alpha - \tau$ space that illustrate this result.

Figure 2: Regions of Uniqueness for Type-2 Rules

$$\tau = 0.5$$



The general result, that a unique REE exists when the monetary authority uses a rule expressed in terms of an interest rate of maturity other than one period, is surprising in light of the intuition previously mentioned. Consider, for example, the case in which the central bank uses a type-2 rule based on the two-period interest rate. One might initially believe that a unique equilibrium would not arise

in this case since the expectations hypothesis of the term structure suggests that infinite combinations of the short-term rate would satisfy the central bank's setting of R_2 . In fact, recall that the expectations hypothesis says that the two-period rate is determined by $R_{2,t} = \frac{1}{2}(R_{1,t} + E_t R_{1,t+1})$. At first sight, one chosen value for $R_{2,t}$ could be achieved by infinitely many paths for R_1 , so that uniqueness could not be achieved.

Clearly, the results show that this is not the case in large and plausible regions of the parameter space. To understand this result, recall that the expectations hypothesis of the term structure says that long-term rates are determined by the expected future path of the short-term rate during the maturity horizon in question. Thus, it makes sense to think that short-term rates determine the level of long-term rates. However, it is important to realise that the expectations hypothesis works in the opposite direction as well. To see this formally, rewrite the equation for R_2 as a first-order stochastic difference equation in R_1 :

$$R_{1,t} = 2R_{2,t} - E_t R_{1,t+1}$$

Advance the equation one period and substitute it back to obtain

$$R_{1,t} = 2R_{2,t} - 2E_t R_{2,t+1} + E_t R_{1,t+2}$$

Repeating this operation many times and using the fact that $\lim_{j \rightarrow \infty} E_t R_{1,t+j} = 0$ yields¹⁸,

$$R_{1,t} = 2 \sum_{j=0}^{\infty} (-1)^j E_t R_{2,t+j} \quad (10)$$

Equation (10) uncovers why the previous intuition is incorrect. A uniquely expected path for the two-period rate, as given by Equation (10), determines a unique current level of the one-period rate. It can be shown that the general expression of the relevant path to be followed for an interest rate of maturity i in order to determine the current level of the one-period nominal rate is given by

$$R_{1,t} = i \left(\sum_{j=0}^{\infty} E_t R_{i,t+j} - \sum_{j=0}^{\infty} E_t R_{i,t+j+1} \right) \quad (11)$$

¹⁸ Variables are expressed in percentage deviations from the steady state. So, in a stationary equilibrium the current expectation of a variable that is far into the future would be zero.

Equation (11) generalises the argument for interest rates of any term. Thus, whatever the maturity of the interest rate chosen for the type-2 rule, if the rule implies a unique equilibrium, then the expected path of the interest rate in the rule uniquely determines the current level of interest rates of longer as well as shorter maturities.

The result that uniqueness arises even when the central bank decides to use a Taylor-type policy rule based on an interest rate other than the short-term rate is important for several reasons. On a somewhat subtle level, it provides support for the theoretical study of macro-monetary models at various frequencies without implying any kind of hidden inconsistency. Without taking a stand on whether time is continuous or discrete, it should almost go without saying that real-world economics occurs at (at least) a daily frequency. As I have already mentioned, discrete models in monetary economics are generally studied at a quarterly frequency without an explicit concern for whether or not the theoretical short-term rate (a 3-month rate) implies a unique level of the current overnight rate. Although models are usually studied at a quarterly frequency, sometimes calibration is done at a monthly or annual frequency. The result presented here provides a theoretical foundation for such frequency choices.

Perhaps more importantly, the result opens up a new dimension of analysis for monetary policy rules that is interesting in its own right. Namely, which interest rate, among a given class of rules, performs best? Which rule gets closer to the optimal monetary policy rule in the sense of Giannoni and Woodford (2002)? Section 4 of the paper addresses these questions.

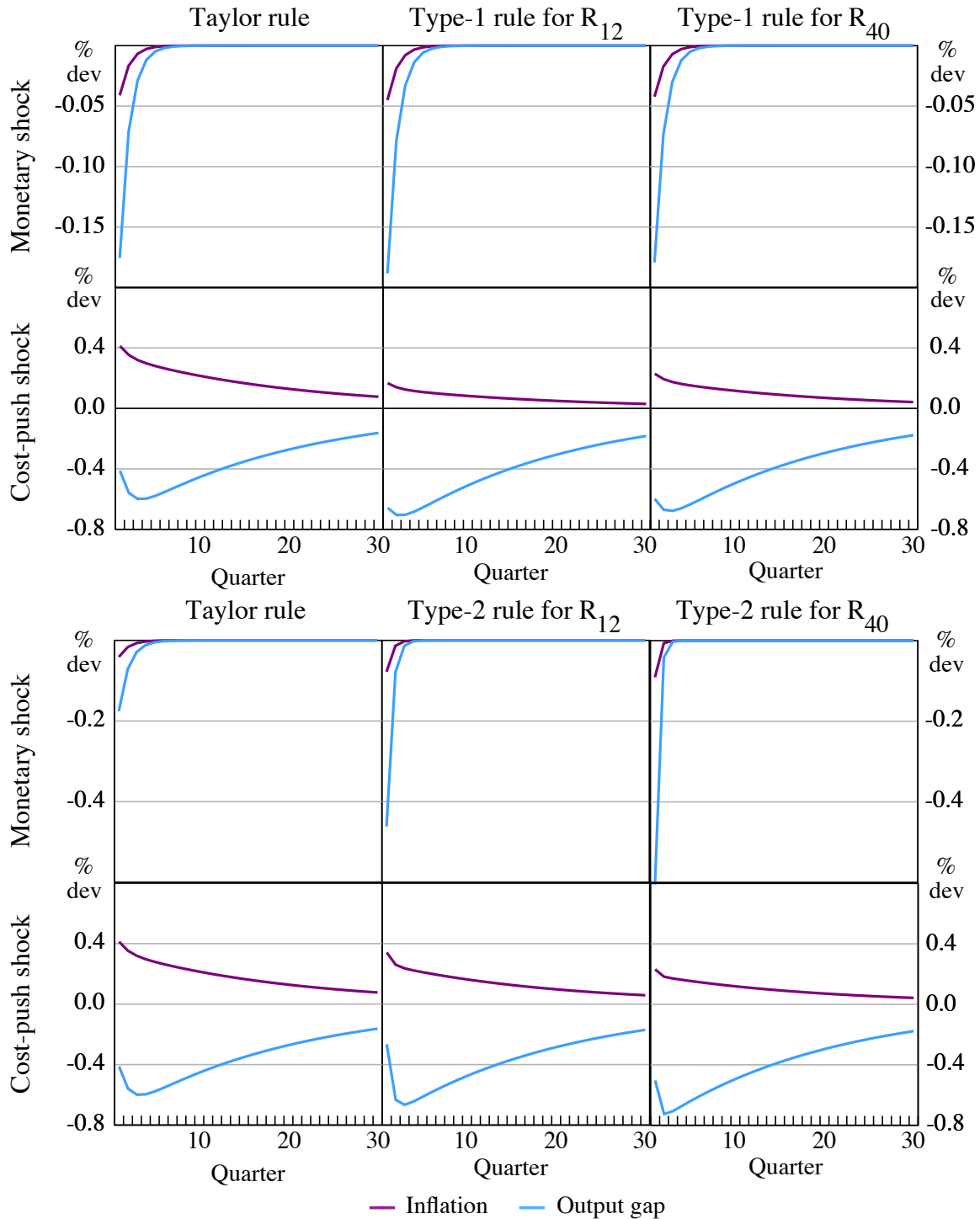
3.3 Dynamics

I have previously shown that large regions of uniqueness exist for type-1 and type-2 rules. It seems interesting to study the dynamic response of the economy to shocks under representative type-1 and type-2 rules versus a standard Taylor rule.

Figure 3 illustrates the impulse responses of inflation and the output gap to a monetary shock and a cost-push shock for a standard Taylor rule, and for type-1 and type-2 rules involving R_{12} and R_{40} respectively. The parameters remain fixed across rules in order to capture the effects implied by the maturity dimension of the problem.

Figure 3: Impulse Responses for Inflation and the Output Gap

$$\tau = 0.6, \alpha = 0.6, \delta = 0.1 \text{ and } \gamma = 0.5$$



A number of interesting features arise from this comparison. First, the signs of the responses do not change across policies. The qualitative responses of inflation and the output gap are the same as those generated by a standard Taylor rule. A contractionary monetary policy shock reduces both inflation and the output gap for all rules considered, while an adverse cost-push shock increases inflation and decreases output for all rules as well. These results suggest that type-1 and type-2 rules do not imply ‘strange’ responses to the disturbances that hit the economy. Secondly, observe that the size of the responses is significantly affected by the type of policy rule. In the case of type-1 rules, in response to a cost-push shock, inflation deviates less than in the Taylor rule case while output suffers a bigger contraction. A similar result shows up in the case of type-2 rules in response to an adverse cost-push shock. In this case, the maturity length of the interest rate in the type-2 rule matters for the determination of the trade-off between output and inflation deviations from the steady state.

Finally, Figure 3 shows that a contractionary monetary shock implies a bigger contraction for inflation and output the longer the maturity of the interest rate in the rule. This arises because the size of the monetary shock changes across policy rules. That is, a one-standard deviation of b_t attached to a type-2 rule for R_{40} raises the one-period nominal interest rate by more than a one-standard deviation of b_t in a standard Taylor rule. Hence, the size of the monetary policy shock, μ_b , is set to zero in all policy rules for the normative analysis of Section 4.¹⁹

As shown, the impulse responses suggest that the dynamic behaviour of the economy is significantly affected by the choice of policy rule. In fact, different rules imply a distinct trade-off between inflation and output deviations in response to a cost-push shock. This shock plays a key role in the conduct of monetary policy. It presents the monetary authority with a trade-off between output and inflation stabilisation. The fact that quantitatively different responses are observed under different rules motivates the question of what is the preferred monetary policy rule. The next section addresses this question.

¹⁹ This guarantees that the comparison between policy rules is fair in the sense of capturing only the impact of the deterministic component of the rules.

4. Optimal Monetary Policy Rules

I start by constructing an optimal policy rule following Giannoni and Woodford (2002). I consider a rule that would bring about the optimal response to shocks as well as yield a unique stationary equilibrium for the economy. I assume that the objective of the monetary authority is to minimise the expected value of a loss criterium given by

$$J = \frac{1}{2} E_0 \left(\sum_{t=0}^{\infty} \beta^t L_t \right) \quad (12)$$

where the bank's discount rate β is the same as in Equation (3) and the period loss function is of the form

$$L_t = \pi_t^2 + \omega_x x_t^2 + \omega_R R_{1,t}^2 \quad (13)$$

Here, the parameters ω_x and ω_R (assumed to be positive) govern the relative concern for output and short-term nominal interest rate variability. The monetary authority faces the problem of minimising the loss function given by Equation (12) subject to the New IS and Phillips curves given by Equations (1) and (3), respectively. Notice that minimisation is achieved by choosing a time path for $\{R_{1,t}\}_{t=0}^{\infty}$ that minimises the monetary authority's objective and simultaneously satisfies the model's structural equations at each point in time. The first-order conditions of the problem are given by

$$\begin{aligned} \pi_t - \beta^{-1} \Lambda_{1,t-1} + \Lambda_{2,t} - \Lambda_{2,t-1} &= 0 \\ \omega_x x_t + \Lambda_{1,t} - \beta^{-1} \Lambda_{1,t-1} - \lambda \Lambda_{2,t} &= 0 \\ \omega_R R_{1,t} + \Lambda_{1,t} &= 0 \\ R_{1,t} - E_t x_{t+1} + x_t - E_t \pi_{t+1} - \mu_g (1 - \phi) g_t + (1 - \rho) a_t &= 0 \\ \pi_t - \lambda x_t - \beta E_t \pi_{t+1} - \mu_v v_t &= 0 \end{aligned}$$

where $\Lambda_{1,t}$ and $\Lambda_{2,t}$ stand for the Lagrange multipliers associated with the IS and New Phillips curve equations respectively. One can use the equations above to substitute out the Lagrange multipliers in order to obtain a monetary policy rule consistent with the optimal solution of the form

$$R_{1,t} = \left(1 + \frac{\lambda}{\beta}\right) R_{1,t-1} + \frac{1}{\beta} \Delta R_{1,t-1} + \frac{\lambda}{\omega_R} \pi_t + \frac{\omega_x}{\omega_R} \Delta x_t \quad (14)$$

As shown by Giannoni and Woodford (2002), commitment to this rule implies a unique equilibrium as well as an optimal pattern of responses to the economy's disturbances. In this case, the optimisation is not performed over some parametric set of policy rules (for example, a Taylor-type rule). Instead, the approach underlying Equation (14) characterises the optimal response to shocks by taking the structural equations as constraints, and then finds the policy rule that generates such an equilibrium.

Any rule constrained to belong to a given set of rules cannot perform better than this optimal rule. Hence, type-1 or type-2 rules could not possibly yield a better outcome than Equation (14). However, studying how close these rules come to the optimal response remains important for many reasons. McCallum's (1988) critique, namely that the main problem that policy-makers face is uncertainty about the exact structure of the economy, implies uncertainty about the exact specification of the optimal monetary policy rule. For this reason it remains important to understand how a given rule works across different plausible environments. Simple policy rules have also been proposed on the basis of being operational and simple to communicate to the public. Operational considerations suggest that rules should be expressed in terms of instrument variables that can be controlled by central banks and require only information available to central banks.²⁰ Recall that the outcome of any type-2 rule can be reproduced by some rule for the short-term rate. So implementation of type-2 rules need not require modifications of operating procedures.

I therefore consider the optimal rule, Equation (14), as a benchmark to evaluate the performance of type-1 and type-2 rules (restricting attention to those that result in a unique REE). For reference, I also examine outcomes based on the standard Taylor rule.

²⁰ If the central bank collects data with a lag, then depending on the frequency of the model, the rules described here might not be operational in practice. Notably, McGough *et al* (2005) address this problem and find regions of uniqueness for policies that set a longer-term rate in response to lagged inflation.

4.1 Type-1 Rules

Table 1 shows, for different calibrations of ω_x and a value of 0.1 for ω_R , the value of the loss function and each of its components for the optimal rule (Equation (14)), the standard Taylor rule (Equation (4)), and type-1 rules of the selected term structure.²¹

Table 1: Optimal Type-1 Rules							
$\omega_R = 0.1$							
	Optimal rule	Taylor rule	R_2	R_4	R_{12}	R_{20}	R_{40}
$\omega_x = 0.1$							
<i>Loss</i>	0.9926	1.0032	1.0032	1.0032	1.0032	1.0032	1.0032
<i>Var(π)</i>	0.1015	0.1147	0.1147	0.1147	0.1147	0.1147	0.1147
<i>Var(x)</i>	8.8961	8.8824	8.8824	8.8824	8.8824	8.8824	8.8824
<i>Var(R_1)</i>	0.0150	0.0031	0.0031	0.0031	0.0031	0.0031	0.0031
$\omega_x = 0.33$							
<i>Loss</i>	2.8191	2.8744	2.8744	2.8744	2.8744	2.8744	2.8744
<i>Var(π)</i>	0.4574	0.3616	0.3614	0.3614	0.3614	0.3614	0.3614
<i>Var(x)</i>	7.0625	7.5377	7.5383	7.5383	7.5382	7.5383	7.5383
<i>Var(R_1)</i>	0.0757	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027
$\omega_x = 1$							
<i>Loss</i>	6.7243	7.5900	7.5900	7.5900	7.5900	7.5900	7.5900
<i>Var(π)</i>	1.6088	0.8299	0.8299	0.8301	0.8299	0.8299	0.8299
<i>Var(x)</i>	5.0405	6.7488	6.7488	6.7486	6.7488	6.7488	6.7488
<i>Var(R_1)</i>	0.7504	0.1130	0.1130	0.1130	0.1130	0.1130	0.1130
$\omega_x = 3$							
<i>Loss</i>	14.2980	19.5110	19.5110	19.5110	19.5110	19.5110	19.5110
<i>Var(π)</i>	4.8407	3.7726	3.7726	3.7725	3.7726	3.7726	3.7726
<i>Var(x)</i>	3.0422	5.1710	5.1710	5.1711	5.1710	5.1710	5.1710
<i>Var(R_1)</i>	3.3090	2.2525	2.2525	2.2524	2.2525	2.2525	2.2525

²¹ Notice that $\frac{1}{2}E_0 \sum_{t=0}^{\infty} \beta^t (\pi_t^2 + \omega_x x_t^2 + \omega_R R_{1,t}^2) = \frac{1}{2(1-\beta)} (\sigma_\pi^2 + \omega_x \sigma_x^2 + \omega_R \sigma_{R_1}^2)$. So minimising J is equivalent to minimising a weighted average of the variances as given by $\frac{1}{2(1-\beta)} (\sigma_\pi^2 + \omega_x \sigma_x^2 + \omega_R \sigma_{R_1}^2)$. Since $\frac{1}{2(1-\beta)}$ is only a scaling constant, one can focus on the value of $\hat{J} = \sigma_\pi^2 + \omega_x \sigma_x^2 + \omega_R \sigma_{R_1}^2$ instead. This is the value of the loss function reported in all tables.

Reacting to movements in nominal longer-term rates does not present significant gains. As expected, type-1 rules are no worse than the standard Taylor rule, since this rule is a particular case of a type-1 rule for which γ is set to zero. Inspection of Table 1 shows not only that type-1 rules achieve a value of the loss function which is identical to that of the Taylor rule, but also that the variances of the relevant variables remain unchanged.²²

In general, for all calibrations and maturities considered, the optimal value of γ turns out to be negative. This is a surprising result in light of Goodfriend's (1993) account of monetary policy and Mehra's (1999) econometric results. Recall that the rationale for allowing the monetary authority to react to movements in long-term bond yields is that long-term nominal interest rates could measure the private sector's long-term inflationary expectations. The central bank might, therefore, be interested in using reaction functions that incorporate longer-term rates, so that if they rise the bank raises the short-term rate in its attempts to keep inflation under control. In other words, this behaviour would imply a positive value of γ .²³

Since there appear to be no significant gains in reacting to movements in longer-term rates, I do not investigate any further the behaviour of type-1 rules under alternative calibrations for ω_x and ω_R .

4.2 Type-2 Rules

For different calibrations of ω_x and a value of 0.1 for ω_R , Table 2 shows the value of the loss function and the value of each of its components for the optimal rule (Equation (14)), the Taylor rule (Equation (4)), and type-2 rule (Equation (6)) for interest rates at selected maturities.

²² This is, in general, true up to the 9th decimal digit. A higher numerical precision shows that type-1 rules are better than the standard rule as one would expect. However, this difference is obviously trivial.

²³ Interestingly, there is recent macro-finance literature that includes the long-run expected inflation component of the long-term rate in the policy rule and can justify negative values for γ . For example, in response to a perceived decrease in the inflation target (a decrease in the expected inflation component of the long rate), the monetary authority must increase rates in order to push inflation down to this lower target. This behaviour would justify a negative value of γ . See Rudebusch and Wu (2004).

Table 2: Optimal Type-2 Rules

$\omega_R = 0.1$

	Optimal rule	Taylor rule	R_2	R_4	R_{12}	R_{20}	R_{40}
$\omega_x = 0.01$							
<i>Loss</i>	0.1167	0.1180	0.1179	0.1177	0.1182	0.1189	0.1195
<i>Var</i> (π)	0.0039	0.0032	0.0032	0.0032	0.0033	0.0024	0.0012
<i>Var</i> (x)	11.0347	11.2289	11.2218	11.2058	11.1868	11.2659	11.4064
<i>Var</i> (R_1)	0.0252	0.0248	0.0245	0.0244	0.0307	0.0386	0.0423
$\omega_x = 0.05$							
<i>Loss</i>	0.5287	0.5376	0.5374	0.5370	0.5359	0.5362	0.5377
<i>Var</i> (π)	0.0396	0.0443	0.0440	0.0435	0.0426	0.0433	0.0453
<i>Var</i> (x)	9.7381	9.8486	9.8494	9.8505	9.8377	9.8192	9.7952
<i>Var</i> (R_1)	0.0216	0.0094	0.0095	0.0098	0.0138	0.0194	0.0262
$\omega_x = 0.1$							
<i>Loss</i>	0.9926	1.0032	1.0032	1.0029	1.0028	1.0027	1.0035
<i>Var</i> (π)	0.1015	0.1147	0.1143	0.1134	0.1116	0.1097	0.1110
<i>Var</i> (x)	8.8961	8.8824	8.8853	8.8917	8.9075	8.9230	8.9173
<i>Var</i> (R_1)	0.0150	0.0031	0.0031	0.0033	0.0040	0.0063	0.0069
$\omega_x = 0.33$							
<i>Loss</i>	2.8191	2.8744	2.8752	2.8766	2.8773	2.8775	2.8778
<i>Var</i> (π)	0.4574	0.3616	0.3593	0.3562	0.3556	0.3554	0.3515
<i>Var</i> (x)	7.0625	7.5377	7.5472	7.5612	7.5651	7.5662	7.5789
<i>Var</i> (R_1)	0.0757	0.0027	0.0019	0.0006	0.0000	0.0000	0.0000
$\omega_x = 1$							
<i>Loss</i>	6.7243	7.5900	7.6189	7.6732	7.7088	7.7089	7.7105
<i>Var</i> (π)	1.6088	0.8299	0.7823	0.6832	0.5987	0.6066	0.6135
<i>Var</i> (x)	5.0405	6.7488	6.8280	6.9865	7.1101	7.1023	7.0970
<i>Var</i> (R_1)	0.7504	0.1130	0.0872	0.0347	0.0000	0.0000	0.0000
$\omega_x = 3$							
<i>Loss</i>	14.2980	19.5110	19.8466	20.6461	21.8445	21.9581	22.0236
<i>Var</i> (π)	4.8407	3.7726	3.5856	3.1417	5.8654	6.0786	6.1891
<i>Var</i> (x)	3.0422	5.1710	5.3491	5.7716	5.1613	5.1196	5.1001
<i>Var</i> (R_1)	3.3090	2.2525	2.1369	1.8943	4.9512	5.2074	5.3430

When the concern for the output variance is relatively low, type-2 rules based on rates of longer maturities generally perform better than the Taylor rule. For example, when ω_x equals 0.05, all type-2 rules except the one based on R_{40} do better than the Taylor rule, with the R_{12} rule being the best among these. Also, note that a type-2 rule based on R_{12} generates a lower variance for R_1 and higher variances for inflation and output relative to the optimal rule, but lower variances of inflation and output relative to the Taylor rule.

When the concern for output volatility increases to 0.1, the best among the type-2 rules shown is that which is based on the 5-year rate. Inspection of Table 2 reveals that in this case the relevant gain comes from the ability of this R_{20} rule to generate a lower variance of inflation. Observe that when ω_x equals 0.01, 0.05 and 0.1 (that is, when the concern for output deviations is relatively low) the variance of the short-term rate increases with the maturity of the interest rate in the rule. For these parameter values, type-2 rules yield a higher variance of the short-term rate than that of the Taylor rule. However, this property is not preserved when the concern for output deviations increases to $\frac{1}{3}$ or to 1.

Table 3 reproduces the results of Table 2 but this time with ω_R set to 1. With this greater aversion to the variance of the short-term interest rate, the variance of R_1 is reduced in all cases and for all rules. In Table 3, for all values of ω_x considered, the variance of the short-term rate achieved by type-2 rules is lower than it is for the optimal rule case and for the Taylor rule.²⁴

A comparison of Tables 2 and 3 shows that when the concern for output variance is relatively low the optimal type-2 rule (in terms of the maturity of the interest rate) is sensitive to the value of ω_R . For example, when ω_x equals 0.1 and ω_R equals 0.1, the best rule is that based on R_{20} . However, when ω_x equals 0.1 and ω_R equals 1, the best rule is that based on R_{12} . Despite these differences in the maturity length, Tables 2 and 3 show an interesting pattern. When the relative concern for output volatility is low, medium/long-term rate rules perform better than the Taylor rule, and when the concern for the output variance is high, the Taylor rule turns out to be better. In short, the best rule depends upon the parameters determining the preferences of the monetary authority.

²⁴ This is, in principle, an interesting result in light of the liquidity trap problem. If, instead, one found that for all type-2 rules, the variance of the short-term rate increases, then the chance of hitting the zero lower bound would increase under type-2 rules.

Table 3: Optimal Type-2 Rules

$$\omega_R = 1$$

	Optimal rule	Taylor rule	R_2	R_4	R_{12}	R_{20}	R_{40}
$\omega_x = 0.01$							
<i>Loss</i>	0.1280	0.1289	0.1288	0.1287	0.1306	0.1335	0.1366
<i>Var</i> (π)	0.0128	0.0129	0.0128	0.0127	0.0156	0.0199	0.0254
<i>Var</i> (x)	10.9092	10.9984	10.9958	10.9874	10.9132	10.8553	10.7963
<i>Var</i> (R_1)	0.0062	0.0061	0.0061	0.0061	0.0058	0.0050	0.0032
$\omega_x = 0.05$							
<i>Loss</i>	0.5375	0.5419	0.5418	0.5415	0.5415	0.5426	0.5440
<i>Var</i> (π)	0.0479	0.0522	0.0520	0.0515	0.0522	0.0549	0.0584
<i>Var</i> (x)	9.7043	9.7458	9.7472	9.7488	9.7300	9.7017	9.6733
<i>Var</i> (R_1)	0.0044	0.0024	0.0024	0.0026	0.0028	0.0026	0.0018
$\omega_x = 0.1$							
<i>Loss</i>	0.9993	1.0047	1.0046	1.0045	1.0044	1.0048	1.0053
<i>Var</i> (π)	0.1090	0.1183	0.1181	0.1175	0.1166	0.1175	0.1193
<i>Var</i> (x)	8.8679	8.8555	8.8571	8.8607	8.8685	8.8638	8.8538
<i>Var</i> (R_1)	0.0035	0.0008	0.0008	0.0009	0.0010	0.0009	0.0006
$\omega_x = 0.33$							
<i>Loss</i>	2.8492	2.8757	2.8762	2.8769	2.8773	2.8778	2.8780
<i>Var</i> (π)	0.3916	0.3581	0.3572	0.3558	0.3555	0.3509	0.3553
<i>Var</i> (x)	7.3225	7.5506	7.5554	7.5629	7.5656	7.5806	7.5684
<i>Var</i> (R_1)	0.0167	0.0008	0.0005	0.0002	0.0000	0.0000	0.0000
$\omega_x = 1$							
<i>Loss</i>	7.1171	7.6441	7.6601	7.6891	7.7070	7.7098	7.7104
<i>Var</i> (π)	1.1005	0.6977	0.6772	0.6367	0.6058	0.6064	0.6055
<i>Var</i> (x)	5.7595	6.9144	6.9589	7.0434	7.1011	7.1034	7.1049
<i>Var</i> (R_1)	0.2570	0.0320	0.0240	0.0090	0.0000	0.0000	0.0000
$\omega_x = 3$							
<i>Loss</i>	16.3987	20.5962	20.8247	21.3184	21.7777	21.7844	21.7845
<i>Var</i> (π)	3.0671	1.8028	1.6506	1.2557	0.7568	0.7618	0.7485
<i>Var</i> (x)	3.9136	6.0524	6.2104	6.5889	7.0070	7.0075	7.0120
<i>Var</i> (R_1)	1.5907	0.6364	0.5428	0.2960	0.0000	0.0000	0.0000

Tables 2 and 3 reveal the presence of two different kinds of trade-offs between the variance of inflation and the variance of output. The first is the well-known Taylor curve trade-off that arises when the parameter that governs the relative degree of concern for the output gap variance, ω_x , varies. For all rules considered, as ω_x rises from 0.01 to 3, the central bank generates a lower variance for output at the expense of a higher one for inflation.

The second trade-off between the variance of output and the variance of inflation, which to the best of my knowledge is new in the literature, shows up in terms of maturities. Notice that a trade-off between the output and inflation variance appears for given values of ω_x and ω_R as we move along the maturity dimension of the class of type-2 rules. However, this trade-off is not working in the same direction in all cases. For example, it is not always the case that when the maturity length of the type-2 rule increases, one observes a lower variance of inflation at the expense of a higher one for output. Figure 4 illustrates this by plotting the loss function and its components against the maturity of the interest rate in the rule. Panel A plots the case when ω_x equals 0.1 and ω_R equals 0.1. As the maturity of the interest rate rule increases up to 20, the variance of inflation decreases and the variance of output increases, but as we move further along the term structure the trade-off changes direction. Panel B shows the case when the preferences of the central bank are ω_x equals 3 and ω_R equals 0.1. In this case the variance of inflation decreases and that of output increases but only up to a maturity of four quarters. The direction of the trade-off changes above this maturity. It is important to emphasise that this is a trade-off that emerges for given values of the parameters that govern the preferences of the monetary authority, ω_x and ω_R . Also, the precise nature of this trade-off changes when the preferences of the central bank change.

One further feature of type-2 rules is worth highlighting. Compared to the standard Taylor rule, they appear to be more forgiving of deviations from their optimal setting. An example of this is provided in Figure 5, which shows the behaviour of the loss function as we depart from the optimal value of one of the parameters in the policy rule (holding the other parameters at their optimal values). For this particular set of preferences the Taylor rule happens to outperform type-2 rules. Even so, the loss function appears to be flatter for type-2 rules (across a wide range of parameter values). What Figure 5 shows is that it seems relatively less costly for the central bank to deviate from the optimal values of τ , α , and δ using the type-2 rule for R_{12} , as opposed to deviations from the best form of the Taylor rule (or

even the type-2 rule for R_{40}). This flatness of type-2 rules is potentially beneficial for policy-makers with some uncertainty about the optimal parameters of a rule.

Figure 4: Optimal Policy for Type-2 Rules – Maturity Trade-off

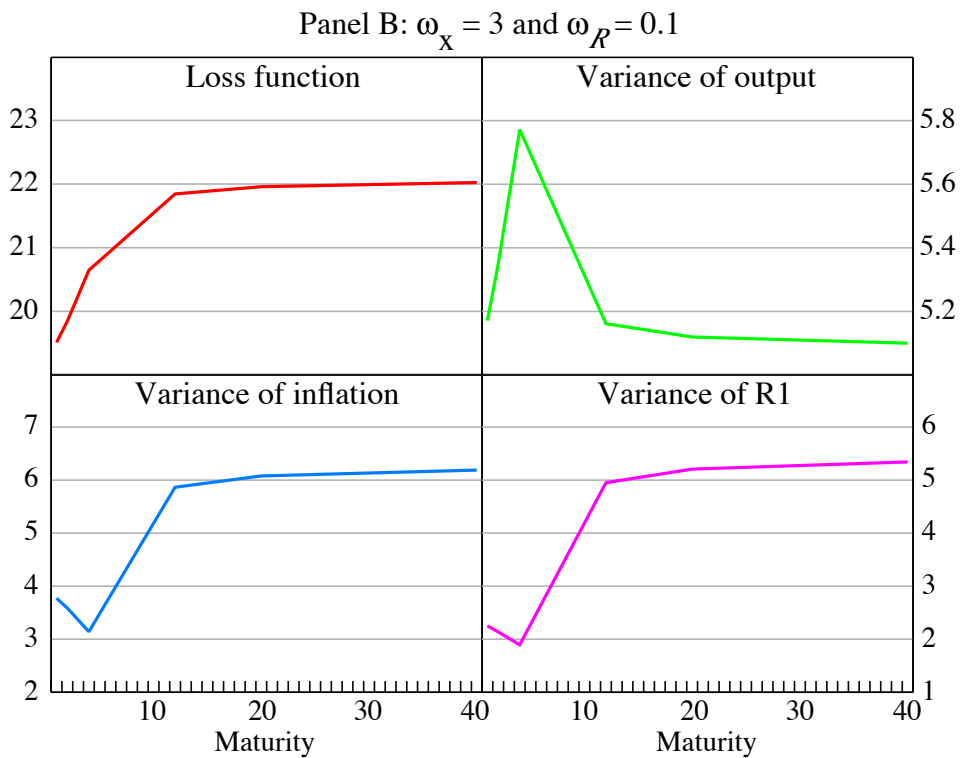
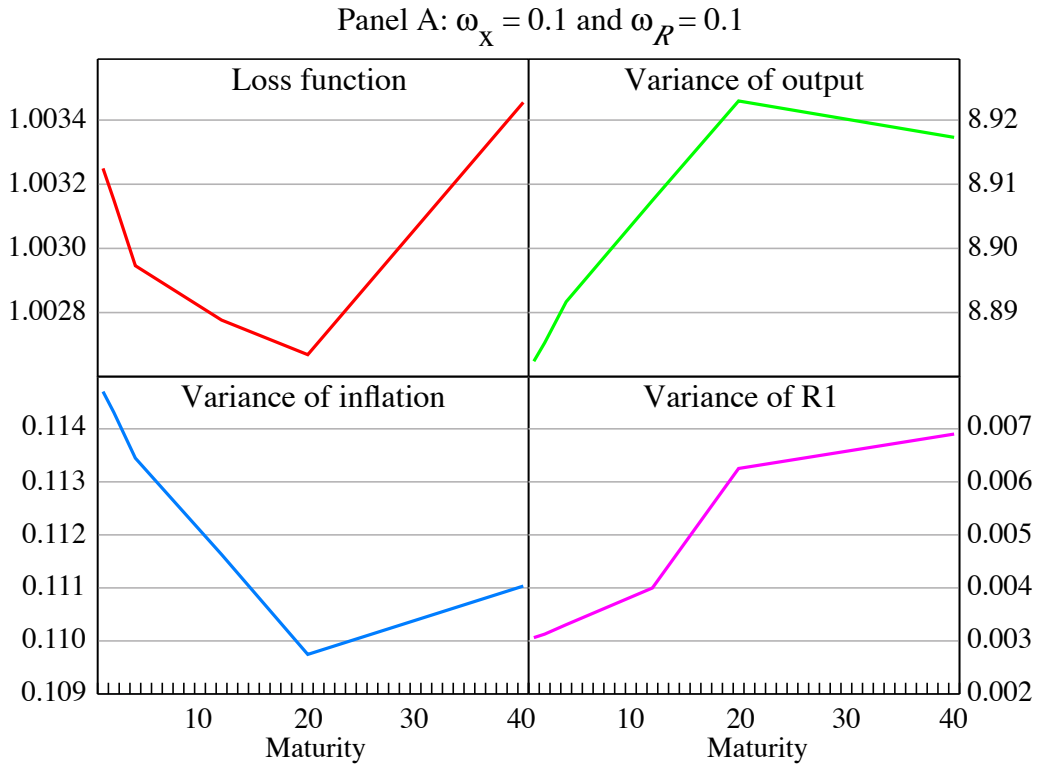
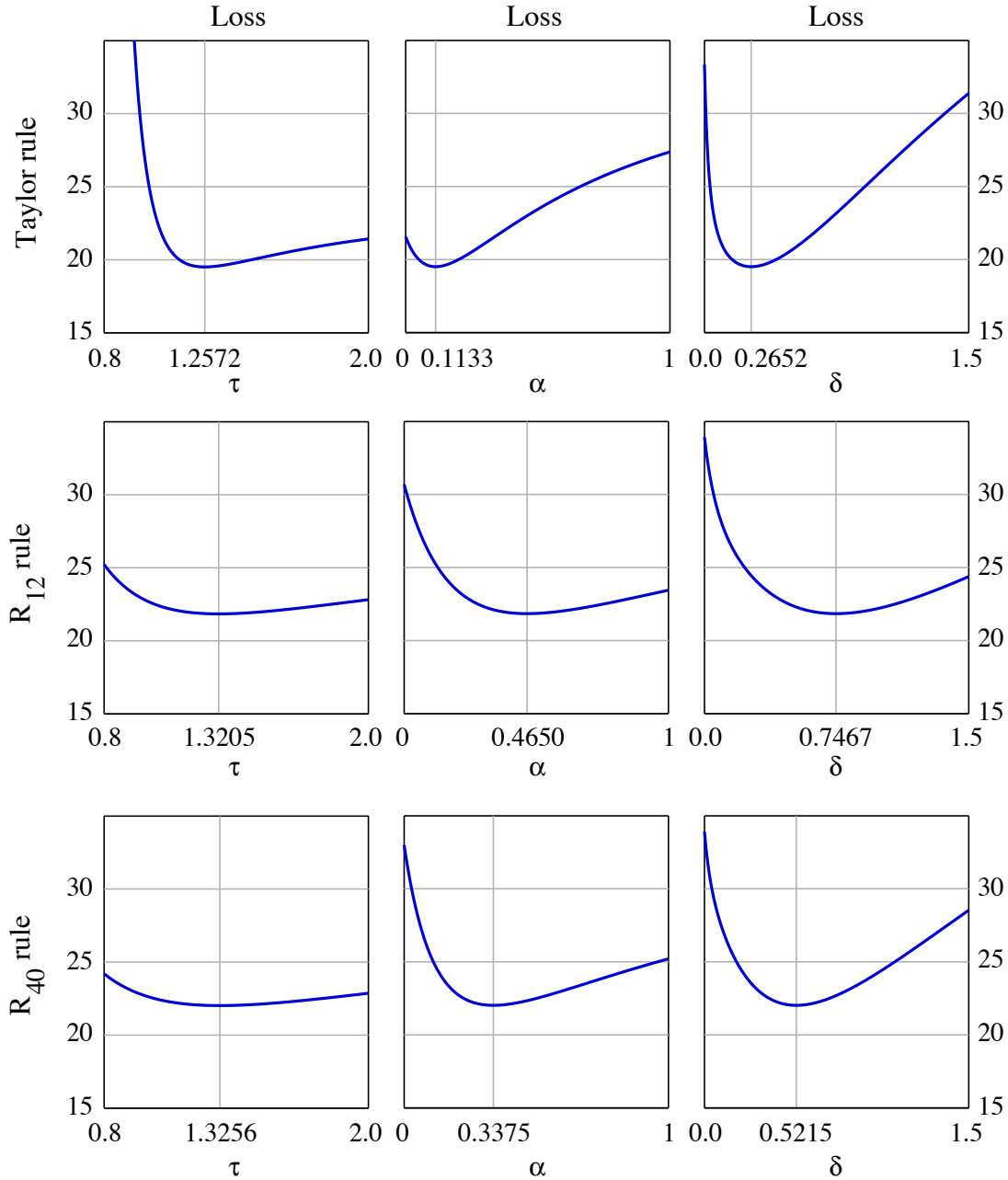


Figure 5: Comparing Type-2 Rules

$$\omega_x = 3 \text{ and } \omega_R = 0.1$$



5. Conclusion

In this paper I have studied the implications of using long-term nominal interest rates in two types of monetary policy rules. Under the first, type-1 rules, the monetary authority adjusts the short rate in response to movements in some long-term yields as well as output and inflation. There are plausible regions of

the policy-parameter space for which a unique stationary REE arises under such policy rules. However, normative analysis reveals that in the context of a simple New Keynesian model there are no significant gains from using type-1 rules in terms of reducing the value of the loss function. Surprisingly, the optimal parameter value of the reaction to long-term rates turns out to be negative for a range of plausible preferences, contradicting the initial intuition that recommends such use.

Under the second use, the monetary authority conducts policy according to a type-2 rule, which is like a Taylor rule but with the short rate replaced by a longer-term rate. Mathematically this is equivalent to a more complicated rule for the short-term rate constructed such that the long-term rate would move in accordance with a Taylor rule. There are a number of surprising aspects of this proposal. First, significant regions of the policy-parameter space exist where a unique stationary REE obtains. Those policy parameters that yield a unique REE can be characterised as satisfying a generalised version of the Taylor principle – namely, that the long-run reaction of the instrument to movements in inflation should exceed one.

Second, type-2 rules can be shown to be better under certain central bank preferences than the standard Taylor rule. In particular, when the relative concern for output variability is relatively low, medium or long-term interest rate rules turn out to yield a better outcome. That is, the choice of maturity length for the rule is sensitive to the preferences of the central bank.

Third, even when preferences are such that optimal use of a Taylor rule outperforms type-2 rules, the latter seem to be more forgiving of ‘mistakes’ in setting the parameter values of the rule.

It is worth noting that these results hold under the pure expectations hypothesis (PEH) of the yield curve. This is not to say, however, that type-2 rules would not be preferable to Taylor rules when the PEH does not hold. Indeed, it may be that both type-1 and type-2 rules provide a useful way to respond to important additional information in the yield curve in a way that is not achieved by standard Taylor rules when the PEH fails. Such a possibility is left for future research.

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