THE FAILURE OF UNCOVERED INTEREST PARITY: IS IT NEAR-RATIONALITY IN THE FOREIGN EXCHANGE MARKET?

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ABSTRACT

A risk-averse US investor adjusts the shares of a portfolio of short-term nominal domestic and foreign assets to maximize expected utility. The optimal strategy is to respond immediately to all new information which arrives weekly. We calculate the expected utility foregone when the investor abandons the optimal strategy and instead optimizes less frequently. We also consider the cases where the investor ignores the covariance between returns sourced in different countries, and where the investor makes unsystematic mistakes when forming expectations of exchange rate changes.

We demonstrate that the expected utility cost of sub-optimal behaviour is generally very small. Thus, for example, if investors adjust portfolio shares every three months, they incur an average expected utility loss equivalent to about 0.16% p.a. It is therefore plausible that slight opportunity costs of frequent optimization may outweigh the benefits.

This result may help explain the failure of uncovered interest parity.

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THE FAILURE OF UNCOVERED INTEREST PARITY: IS IT NEAR-RATIONALITY IN THE FOREIGN EXCHANGE MARKET?

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1. INTRODUCTION

Uncovered interest parity is one of the linchpins of modern exchange rate theory. It follows from the joint hypothesis that the foreign exchange market is efficient, that traders are risk-neutral and that transaction costs are negligible. However, uncovered interest parity is overwhelmingly rejected by empirical evidence.[1]

There are four possible interpretations of the failure of this joint hypothesis. The first, and most widely accepted, is that there is a time-varying risk premium[2] required to hold a portfolio of assets denominated in a range of currencies. A model which incorporates risk premia and performs well empirically has, however, proven elusive (see for example, Hodrick (1987), Cumby (1988) and Baillie and Bollerslev (1990)). Furthermore, theory-based estimates of risk premia turn out to be very small indeed (see Frankel (1985), Frankel (1988) and Smith and Gruen, (1989)).

A second possibility is that the joint hypothesis fails because of a small sample bias or 'peso problem'. The suggestion here is that investors rationally estimate the ex ante probabilities of events which, if they occur, will have a significant impact on the real return from their portfolio. During the period under study, if the events do not occur with a frequency consistent with their estimated probabilities, it will incorrectly appear that the investor's behaviour was irrational.

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[1] See Hodrick (1987) and Goodhart (1988) for international evidence with a range of currencies, or Smith and Gruen (1989) for the Australian/US exchange rate.

[2] The term 'risk premium' is often used loosely to mean the excess return demanded by investors to compensate them for the 'risk' of an exchange rate depreciation. We use the term in its technical sense. For a given expectation of the return on an asset, the risk premium is the excess return required because of the expected volatility of the return and its expected correlation with the returns on other assets.

A third possibility (Baldwin (1990)) is that small transaction costs combined with uncertainty can lead to an interest rate differential matched neither by a risk premium nor by an expected exchange rate change. Interest rate differentials within a small band do not set in motion the capital flows that would close the gap because transaction costs render the moving of capital sub-optimal.

The final possible interpretation of the rejection of uncovered interest parity is that the foreign exchange market is not efficient. This is the interpretation we explore in this paper.

We consider the returns available to a representative risk-averse fund-manager who maximizes a function of mean and variance of end-period wealth. The investor chooses the shares held in different currencies by examining interest rates and expected changes in the exchange rate. We discover that, in terms of expected utility, the investor loses very little by exhibiting certain types of 'near-rational' behaviour (see Akerlof and Yellen (1985) for a definition of near-rational). If we then allow for the possibility of small transaction costs, the benefits of being fully rational are even smaller.

Initially, we assume that the investor responds immediately to all new information which affects expected utility. We assume that new information arrives weekly. We then explore three deviations from this strategy.

In the text of the paper, we estimate the cost of changing the portfolio shares infrequently rather than every time new information becomes available. In the Appendix, we examine two further examples of near-rationality. In these examples we assume that investors either:
(i) take account of the expected returns available in different countries and the variability of those returns but ignore the covariances between them, or
(ii) make small mistakes in forming expectations of exchange rate changes.

We estimate the expected utility cost of each of these types of near-rational behaviour using interest rate and exchange rate data over the period 1983 - 1989. The main contribution of the paper is to show that the cost of sluggishly adjusting portfolio shares over periods of relevance to tests of uncovered interest parity is very small indeed. We conclude that there are no strong grounds for expecting that the agents who engage in such behaviour will be driven from the market. This provides a potential explanation for the failure of uncovered interest parity.

2. THE MODEL

In this section we derive the optimal allocation of foreign assets in a representative US investor's portfolio. We maximize expected utility in the coming week.

The US investor maximizes a function of the mean and variance of end-of-period real wealth. At the beginning of each week, all the information necessary to accurately forecast end-of-week real wealth is available, with the exception of the exchange rate movements over the week. Prices are not a source of uncertainty by assumption. We allow assets denominated in US Dollars, Yen, Deutschemarks, British pounds, Swiss francs and/or Australian dollars to be chosen for the portfolio.

We contrast different strategies by investors within this model framework. The first 'rational' strategy involves the investor maximizing expected utility by choosing portfolio shares of hypothetical weekly securities denominated in the available currencies. Relevant new information arrives weekly, and so this 'rational' investor adjusts portfolio shares every week. At the end of each week, the one-week securities mature and new ones are purchased. We assume there are no costs associated with these transactions. Because all the securities are turned over each week, the cost of sub-optimal behaviour arises from holding inappropriate shares of assets, rather than from being 'locked-in' at old interest rates (as would be the case if the securities had a term of more than a week).

We now posit an expected utility function and maximize it with respect to the portfolio shares. The analysis follows Frankel and Engel (1984). We use the following notation:

1	–	a vector of ones
p^{us}	–	US consumer prices
r^j	–	the real return in country j
s^j	–	the exchange rate (value of currency j per $US)
U	–	utility
W	–	real wealth
x^j	–	share of portfolio held in foreign country j

z^j	–	the real return in foreign country j relative to the US
i^{jl}	–	nominal long interest rate in country j (expressed per week)
i^{js}	–	nominal short interest rate in country j (expressed per week)
ρ	–	the coefficient of relative risk aversion
Ω	–	the variance-covariance matrix of real returns relative to the US.

End-week real wealth is given by

$$W_{t+1} = W_t + W_t\, x_t^{'}\, r_{t+1} + W_t(1-x_t^{'}1)r_{t+1}^{us}. \tag{1}$$

x_t is a 5x1 vector of foreign currency shares, x_t^j, chosen at time t, while r_{t+1} is a 5x1 vector of real returns available to the US investor by investing in those countries. The individual elements of r_{t+1} are given by:

$$r_{t+1}^j = (1+i_t^{js})\,\frac{s_t^j}{s_{t+1}^j}\,\frac{p_t^{us}}{p_{t+1}^{us}} - 1.$$

r_{t+1}^{us} is similarly defined, without the exchange rate ratio.

z_{t+1} is a 5x1 vector of real returns relative to the US, given by

$$z_{t+1} = r_{t+1} - 1\, r_{t+1}^{us}\ ,$$

and hence (1) becomes

$$W_{t+1} = W_t\, [x_t^{'} z_{t+1} + 1 + r_{t+1}^{us}]\ .$$

The mean and variance of W_{t+1} are

$$\mu_{t+1} \equiv E_t(W_{t+1}) = W_t\, [x_t^{'}E(z_{t+1}) + 1 + r_{t+1}^{us}] \qquad \text{and}$$

$$\begin{aligned} V_t(W_{t+1}) &= W_t^2\, [x_t^{'}\, \Omega\, x_t + V_t\, r_{t+1}^{us} + 2x_t^{'}\text{cov}_t\, (z_{t+1}, r_{t+1}^{us})] \\ &= W_t^2\, x_t^{'}\, \Omega\, x_t. \end{aligned}$$

The last step follows from the fact that $E(r_{t+1}^{us}) = r_{t+1}^{us}$; by investing in the US the investor removes the only source of uncertainty – exchange rate movements.

The variance-covariance matrix of relative real returns, Ω, is

$$\Omega = E_t(z_{t+1} - E_t(z_{t+1}))\,(z_{t+1} - E_t(z_{t+1}))'.$$

In order to calculate $E_t(z_{t+1})$ we require a simple and tractable expression for the expectation of exchange rate changes. Unit-root tests can rarely reject the hypothesis that nominal exchange rates follow a random walk with no drift (Frankel and Meese, 1987) – which would imply that $E_t\left(\frac{s_t^j}{s_{t+1}^j}\right) = 1$.

However, these tests are of low power, and they can rarely reject a range of alternative hypotheses.

From the perspective of economic theory it seems plausible that, over time, nominal exchange rates move to offset differences in inflation. We use the difference in long bond rates as a proxy for the expected inflation difference, and assume that,

$$E_t\left(\frac{s_t^j}{s_{t+1}^j}\right) = 1 + i_t^{usl} - i_t^{jl}. \tag{2}$$

Equation (2) implies that each real exchange rate is expected to follow a random walk with no drift.[3]

Investors maximize expected utility which is assumed to be a linear combination of the expected value and variance of real wealth. Thus,

$$E_t(U_{t+1}) = \mu_{t+1} - \frac{\rho}{2W_t} V_t(W_{t+1}), \tag{3}$$

where ρ is the coefficient of relative risk aversion evaluated at μ_{t+1}.[4]

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[3] Note that equation (2) implies a rejection of uncovered interest parity because the expected exchange rate change is **not** determined by the short-term interest differential. The point of the paper is to estimate how much is lost by not taking full advantage of the gap between the expected exchange rate change and the short-term interest differential. The Discussion section examines an alternative to the assumption that the real exchange rate follows a random walk.

[4] Equation (3) embodies the approximation that $\rho \equiv -\mu_{t+1} \cdot U''(W_{t+1})/U'(W_{t+1}) \approx -W_t \cdot U''(W_{t+1})/U'(W_{t+1})$. This approximation makes the algebra simpler and is empirically harmless since $1 < \mu_{t+1}/W_t < 1.01$.

Maximizing (3) with respect to x_t, leads to [5]

$$x_t = (\rho\Omega)^{-1}\, E_t(z_{t+1}). \tag{4}$$

This is the key equation which relates the optimal portfolio shares to the mean and variance-covariance matrix of the relative returns.

(a) Introducing constraints on shares

The optimal foreign shares derived from (4) are unconstrained; they can be negative and they can sum to more than one (implying a negative share for the US). A negative share signifies borrowing, say, US dollars to place in a high-yielding low-variance currency.

We now wish to limit the foreign exposure of the portfolio and to impose the constraint that investors don't sell short in a currency – that is, that all shares are non-negative. This is one way to introduce the real-world asymmetry that borrowing and lending rates are not equal.[6] Imposing these additional constraints, the maximization problem is reformulated as follows:

$$\text{maximize } E_t(U_{t+1}) = W_t[x_t' E_t(z_{t+1}) + 1 + r^{us}_{t+1} - \frac{\rho}{2} x_t'\Omega x_t] \tag{5}$$

$$\text{subject to } x_t' 1 \leq k \tag{6}$$

$$\text{and} \qquad x_t^j \geq 0, \tag{7}$$

where $k \leq 1$ is the maximum foreign exposure of the portfolio.

Any solution of this constrained maximization problem must satisfy the Kuhn-Tucker sufficiency conditions:

(a) $E_t(U_{t+1})$ is a differentiable and concave function of x_t in the non-negative orthant,

(b) the constraint function $(x_t'1)$ is differentiable and convex in the non-negative orthant, and

5 Frankel and Engel (1984) have an additional term here which arises from their assumption of stochastic US prices.

6 By making $E_t(z_{t+1})$ the subject of (4), it becomes apparent that relative returns are a linear function of x_t, i.e., that borrowing and lending rates are assumed to be the same.

(c) the following conditions (omitting the time subscript) hold:

I $\frac{\partial \phi}{\partial x^j} \leq 0$; $x^j \frac{\partial \phi}{\partial x^j} = 0$

II $\lambda \geq 0$; $(k - x'1)\lambda = 0$

where $\phi = x_t' E_t(z_{t+1}) + 1 + r^{us}_{t+1} - \frac{\rho}{2} x_t' \Omega x_t + \lambda(k - x_t' 1)$

and λ is the Lagrange multiplier for constraint (6).

Condition (a) is satisfied because $\frac{\partial^2 E_t(U_{t+1})}{\partial x_t^2} = -\rho\Omega$ and Ω is positive definite. Condition (b) is clearly satisfied. Shares from (4) may satisfy constraints (6) and (7), in which case that is the constrained optimum. Otherwise, we derive a solution by alternately eliminating from the analysis countries with negative portfolio shares, and, if constraint (6) is violated, imposing it with equality. The final solution (a 5x1 vector of non-negative portfolio shares) must satisfy all the Kuhn-Tucker sufficiency conditions for all countries.

(b) The cost of near-rationality

Assume that at time t, strategy α leads the investor to choose the vector of portfolio shares x_t^{α} and hence to derive expected utility $E_t\,(U^{\alpha}_{t+1})$. Further, define c^{α}_{t+1} by the equation

$$E_t\,(U^{\alpha}_{t+1}) = W_t[\,1 + c^{\alpha}_{t+1}\,]. \tag{8}$$

Finally, define c^{α} as the arithmetic average of c^{α}_{t+1} over the sample. For any given strategy α, the average equivalent riskless return forgone (AERRF) is $c^{opt} - c^{\alpha}$, where c^{opt} is the value of c^{α} when α is the optimal strategy.

Evaluating the AERRF allows us to quantify and compare the costs of various sub-optimal strategies. Note that the loss associated with any near-rational strategy consists of having inappropriate portfolio shares – rather than missing out on interest rate changes which may turn out to be favourable.

We wrote a GAUSS program to evaluate the optimal portfolio shares under three regimes. In turn, we assumed that foreign portfolio shares were

(i) unconstrained, and hence given by equation (4), or (ii) constrained to be non-negative and with a maximum foreign exposure of 100% (i.e., $k = 1$ in equation (6)) or, (iii) constrained to be non-negative and with a maximum foreign exposure of 40% (i.e., $k = 0.4$ in equation (6)). In each case, we evaluated the expected utility cost (AERRF) when the investor optimizes every 1, 2, 3, 4, 13, 26 and 52 weeks. By assumption, optimizing every week is fully optimal and so the associated AERRF is zero. The results were derived using 302 weeks of data on short and long-term interest rates, and an empirical estimate of Ω (see the Data Appendix and the Appendix for details). Figure 1 shows the results assuming $\rho = 2$, 10 and 20, as Cecchetti and Mark (1990) suggest that there may be a range of possible values for ρ.[7]

Perhaps the most realistic simulations are those shown in the middle panel of Figure 1, which assume that all portfolio shares (including the US share) are non-negative. Assuming $\rho = 2$, the expected utility cost of adjusting portfolio shares every four weeks averages about 0.05%p.a. while the cost of adjusting every three months (13 weeks) averages about 0.16% p.a. If $\rho > 2$, these costs are even smaller (see Figure 1).

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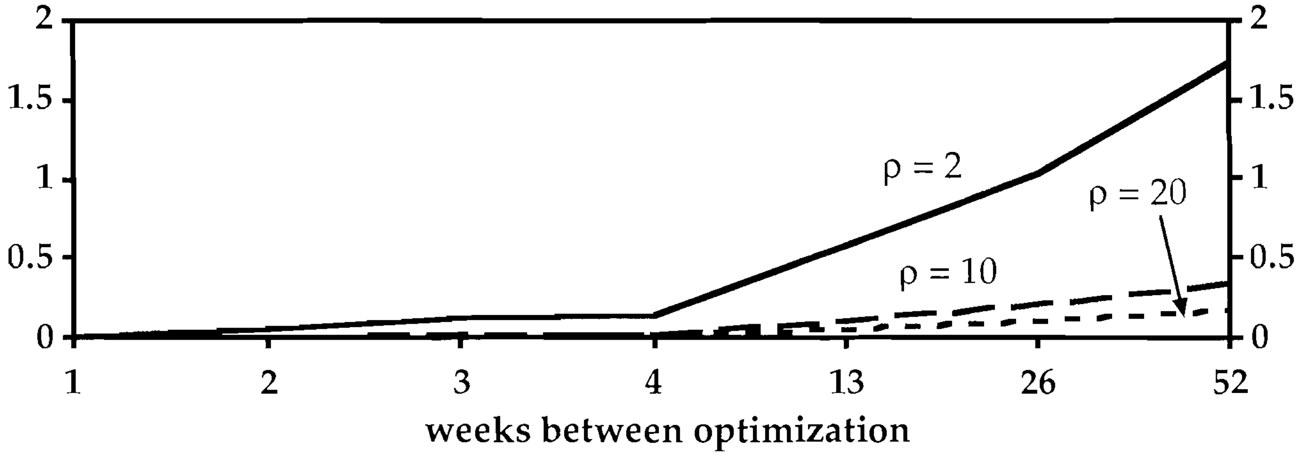
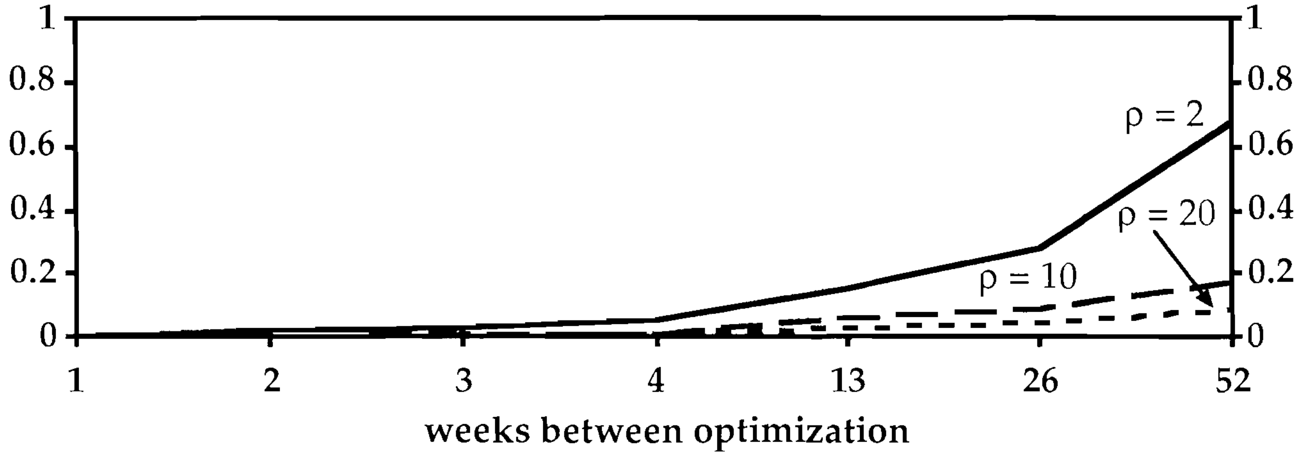
[7] Our enquiries suggest that many fund managers do not hold more than a proportion k (with $k < 1$) of their portfolio overseas – presumably because they are risk-averse. In our framework this implies that $x_t' 1 \leq k$, and manipulation of (4) gives $\rho \geq [(k\Omega)^{-1} E_t(z_{t+1})]'1$. A money market corporation (which practices 'active' portfolio management), an insurance company and a state-government corporation offered upper bounds on the size of the hypothetical investor's overseas portfolio share of 45%, 40% and 20% respectively. $k = 20\%$ (45%) implies $\rho \geq 10$ (4.4) on average over the period. We therefore have some additional grounds for experimenting with values of ρ larger than 2.

Figure 1
COSTS FOR NEAR RATIONAL INVESTORS
(average equivalent riskless return
foregone measured as % p.a.)
ρ = Coefficient of relative risk aversion

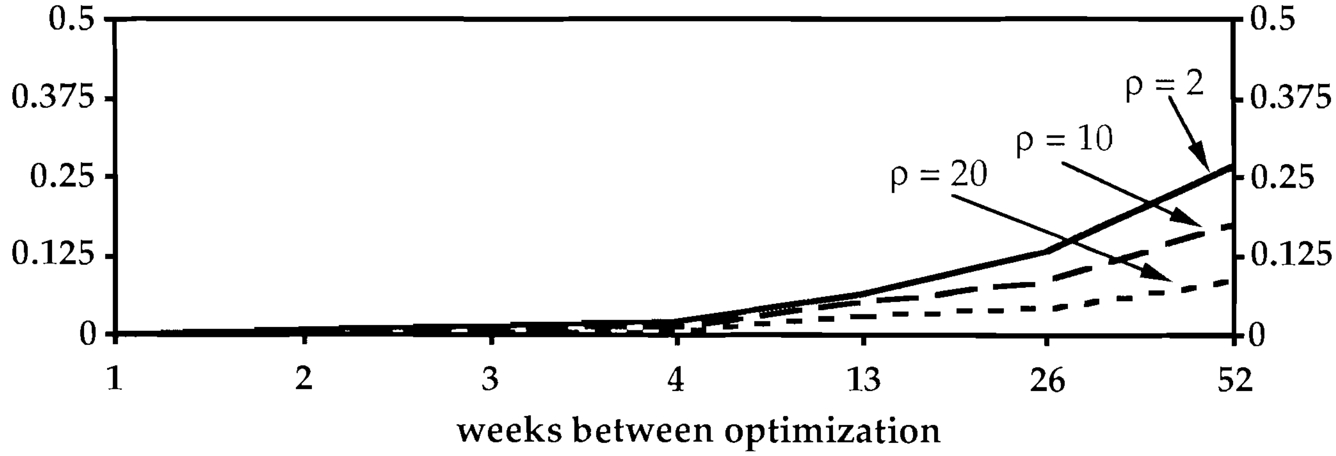
% p.a.
Unrestricted Investor



All Portfolio Shares, Including the US,
% p.a.
Assumed Non-Negative



All Foreign Shares are Non-Negative and must Sum
% p.a.
to No More Than 40%



3. DISCUSSION

The focus of this paper is on the expected utility costs of sub-optimal strategies rather than on ex post outcomes. For periods as long as a year or more, the ex post returns from the near-rational strategies we consider are sometimes **higher** than the returns from the fully-optimal strategy. The following simple argument demonstrates why. Imagine a portfolio restricted to assets in the US and a single foreign country and assume that the expected excess real return in the foreign country is 3%p.a.[8] With $\rho = 2$, and our empirical estimate of the variance of real weekly excess returns sourced in a foreign country from Table 1 of about 0.0003, a mean-variance optimizing investor will put all of her portfolio in the foreign country.[9] We now compare the return from this optimal strategy with the return from the sub-optimal "stay-at-home" strategy of keeping the whole portfolio in the US. By straightforward calculation using equation (3), the expected utility cost of this stay-at-home strategy is about 1.5%p.a.

Over a year, the standard deviation of real returns sourced in the foreign country is about $\sqrt{52}\sqrt{0.0003} \approx 12\%$, while, by assumption, the expected excess return is 3%. Over a year, exchange rate shocks are approximately normally distributed (Baillie and Bollerslev, 1989). Hence, the probability that, in a randomly chosen year, the stay-at-home strategy will produce a higher ex post real return than the optimal strategy is about $\Pr(z > 1/4)$ where z is a standard normal variable, or about 0.4.

Thus, judged by the expected utility derived from it, the stay-at-home strategy is very costly (with an expected utility cost of about 1.5%p.a. compared, for example, with the cost associated with optimizing portfolio shares every three months in our six-currency model of about 0.16%p.a.). Yet, in a randomly chosen year, the stay-at-home strategy outperforms the optimal strategy about 40% of the time. Even over a five-year period, the stay-at-home strategy outperforms the optimal strategy about 29% of the time ($\Pr(z > \sqrt{5}/4) \approx 0.29$).

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[8] This is a substantial expected excess return. In our model framework, it would occur when the short-term differential between foreign and domestic interest rates is 3%p.a. larger than the long-term differential.

[9] Application of equation (4) gives $x_t = (0.0006)^{-1}.\ 0.03/52 \approx 1$.

This simple example demonstrates an important point. Even using five years of data, it is very difficult to distinguish between optimal and near-rational strategies using ex post outcomes.[10] If investors choose to invest their funds with the portfolio manager who has the best performance in, say, the last five years, those managers who engage in near-rational behaviour (e.g., those who optimize their portfolio every three months) will often be chosen as the best performers. Hence, they will not disappear from the market.

The standard test of uncovered interest rate parity is to run the regression

$$\Delta s_{t+\tau} = \alpha + \beta\,(i_t - i_t^*) + \eta_{t+\tau}, \tag{9}$$

where $\Delta s_{t+\tau}$ is the change in the log of the spot domestic price of foreign exchange over τ periods, $(i_t - i_t^*)$ is the current interest differential between τ-period domestic and foreign nominal assets and $\eta_{t+\tau}$ is an error term. Uncovered interest parity holds if $\beta = 1$ in the true model ($\alpha = 0$ is sometimes also included as part of the hypothesis). The overwhelming empirical finding is that $\beta < 1$, and often that $\beta < 0$. To give a representative example, Goodhart (1988) estimates equation (9) for nine data sets. In six cases out of nine, the point estimate for β is negative, and in five cases out of nine, the estimate of β is more than two standard errors less than one. By contrast, in no case is β significantly greater than one.

Almost without exception, empirical rejections of uncovered interest parity use **short-term** nominal interest differentials in equation (9).[11] For example, in the nine tests of uncovered interest parity reported above, six use one month interest rates while the rest use three month interest rates.

Provided some nominal interest rate changes are also real interest rate changes, sluggish adjustment of portfolio shares provides a potential

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[10] It is for this reason that we focus instead on expected utility as a measure of the sub-optimality of the strategies we consider.

[11] By contrast, interest differentials on longer-term nominal assets provide a much better guide to longer-term changes in exchange rates. Countries with relatively high inflation have relatively high long-term nominal interest rates and their currencies tend to secularly depreciate against the currencies of low inflation countries. Thus, the failure of uncovered interest parity is a failure of the exchange rate to move as predicted in the short-term but not in the longer-term. Froot and Thaler (1990) make a similar point.

explanation for the failure of uncovered interest parity. Most exchange rate models (Dornbusch (1976) being the most famous example) predict that, other things equal, an increase in domestic real interest rates leads to an instantaneous appreciation of the domestic exchange rate. With a substantial proportion of investors adjusting sluggishly to changes in interest rate differentials, only part of this appreciation occurs immediately, with the rest occurring over a time scale comparable to the time between portfolio adjustments. Then, on average, the short-term interest differential will over-predict the subsequent change in the exchange rate, i.e., $\beta < 1$. Thus, sluggish adjustment of portfolio shares also provides a potential explanation for the **direction of failure** of uncovered interest parity.

The contribution of this paper is to demonstrate that, in terms of expected utility, the cost of sluggish adjustment of portfolio shares over times relevant to tests of uncovered interest parity is very small indeed. The final step in the argument can then take two possible forms. Either, one can appeal to small transactions or opportunity costs of frequent optimization and argue that sluggish adjustment of portfolio shares is fully optimal. Alternatively, one can argue that, although not fully optimal, sluggish portfolio adjustment costs so little that agents who engage in this behaviour will not be driven from the market.

There are several refinements which could be introduced into our model. Firstly, the conditional variance-covariance matrix for weekly nominal exchange rate changes is more accurately modelled as a GARCH(1,1) process rather than as a time-invariant matrix (Baillie and Bollerslev, 1989) and a similar representation should also be a more accurate model of real excess returns (because the variation in these returns is dominated by nominal exchange rate changes). If Ω follows a GARCH process, there will be some times when the cost of near-rationality is underestimated by our simpler model and other times when it is overestimated. Nevertheless, by an argument similar to Frankel (1988), our model should give a good estimate of the cost of near-rationality averaged over a few months or longer.

Secondly, rather than using equation (2), an alternative modelling strategy would have been to follow Frankel and Meese (1987), and assume that each real exchange rate follows an AR(1) process. Using annual data on the real

US/UK exchange rate during the floating period 1973-1984, Frankel and Meese derive point estimates for the autoregressive coefficient of 0.720, and for the mean absolute deviation of the real exchange rate of 0.121. Using this model would introduce a correction to equation (2) with an **average** magnitude of about (0.121) (1 - 0.720) = 0.034 or 3.4%p.a. While this is a substantial correction, provided this underlying model was known to both the rational and near-rational investors, it is not clear how much difference this refinement would make to the results.

Finally, the analysis is clearly partial equilibrium in nature. There is no examination of how the actions of the representative portfolio investors influence the behaviour of the exchange rates. We intend to examine this link in future research.

DATA APPENDIX

The empirical results in the paper are derived from 302 end-week observations of interest rates and exchange rates from 23 December 1983 to 29 September 1989 inclusive. We use Friday data unless Friday was a holiday, in which case we use the previous trading day's rate.

Exchange Rates

All exchange rates (4 p.m., Sydney) are the foreign currency value of $1US from the International Department of the Reserve Bank of Australia.

Short-Term Interest Rates

3-month interest rates in the Euro-market are chosen as the yield on our hypothetical weekly securities. With the exception of the $A Euro-rate, this data was also provided by the International Department of the Reserve Bank of Australia. The $US, ¥, Swfr and DM rates are London rates. The former two refer to the close while the latter two refer to the afternoon. The £ rate is a Paris reading (time unspecified). From 2 January 1987, the $A rate is the average of the bid and offer rates made available by Deutsche Bank, Sydney (location and time unknown). Before that, the $A rate is the 13-week Australian Treasury Note (mid-day) plus the difference between the $US 3-month Euro-rate and the $US 3-month Treasury Bill (New York close).

Long-Term Interest Rates

Before July 1987, the long-term interest rate data for Japan, West Germany, the United Kingdom and Switzerland are taken from the International Monetary Fund's 'International Financial Statistics' (IFS). The IFS data is monthly, so weekly rates are simply the monthly rate repeated four or five times. Since long bond rates change only gradually, the use of monthly data probably introduces minimal error. Over this period, the definition of the long-rates differs considerably across countries. In West Germany, the rate is the monthly average of all yields on bonds of 3 or more years duration. For Japan, it is the end-month observation on the 7-year bond rate. The UK rate is the Wednesday average over the month of 20-year bond yields, while the Swiss rate is based on yields of 10 or more year bonds.

After June 1987 (and before that in the case of Australia and the US), weekly data on 10-year bond rates are available for all countries except

Switzerland. All the series are end-week observations at the close of business in the relevant financial centres: New York, Tokyo, Frankfurt, London, Zurich and Sydney. The IFS monthly data is used over the whole period for Switzerland.

APPENDIX

1. Estimation of Ω

The variance-covariance matrix of real relative returns, Ω, is defined as $\Omega = E_t(z_{t+1} - E_t(z_{t+1}))\ (z_{t+1} - E_t(z_{t+1}))'$, where $z_{t+1} = r_{t+1} - 1\, r^{us}_{t+1}$. To derive empirical estimates of Ω, we use a sample average of z_t for $E_t(z_{t+1})$. Because US inflation was relatively stable over the sample period, rather than using actual data, we assume that US inflation is fixed at 0.075% per week (equivalent to 4% p.a.). Over a week, exchange rate changes – rather than changes in the US inflation rate – dominate the variance-covariance of relative real returns on a portfolio of short-term nominal assets. Table 1 displays the empirically estimated matrix Ω using our 302 weeks of data.[12]

TABLE 1
THE VARIANCE-COVARIANCE MATRIX, Ω,
OF WEEKLY REAL RETURNS RELATIVE TO THE US

	J	WG	UK	S	A
J	**.000255**	.000215	.000180	.000182	.000074
WG		**.000306**	.000250	.000248	.000069
UK			**.000313**	.000203	.000080
S				**.000398**	.000042
A					**.000248**

12 Prof.W.E.Griffiths pointed out to us that calculating Ω in this manner gives investors the benefit of information they do not yet have. We have established that continually updated estimates of Ω based on available information makes negligible difference to our estimates of the average cost of near-rational strategies.

2. Alternative examples of near-rationality

In this section of the Appendix, we estimate the cost of two further examples of near-rational behaviour: "covariance blindness", and small mistakes in the formation of exchange rate expectations.

To estimate the cost of covariance blindness, we assume that our representative investor uses the covariance matrix from Table 1 with all off-diagonal elements of Ω set to zero when choosing shares.

In our final example of near-rational behaviour, the investor makes an unsystematic error when forming exchange rate expectations. Equation (2) is replaced by:

$$E_t\left(\frac{S_t^j}{S_{t+1}^j}\right) = 1 + i_t^{usl} - i_t^{jl} + e_t^j \qquad (2a)$$

where e_t^j are independent normally distributed errors with zero mean and a standard deviation of 1% p.a.

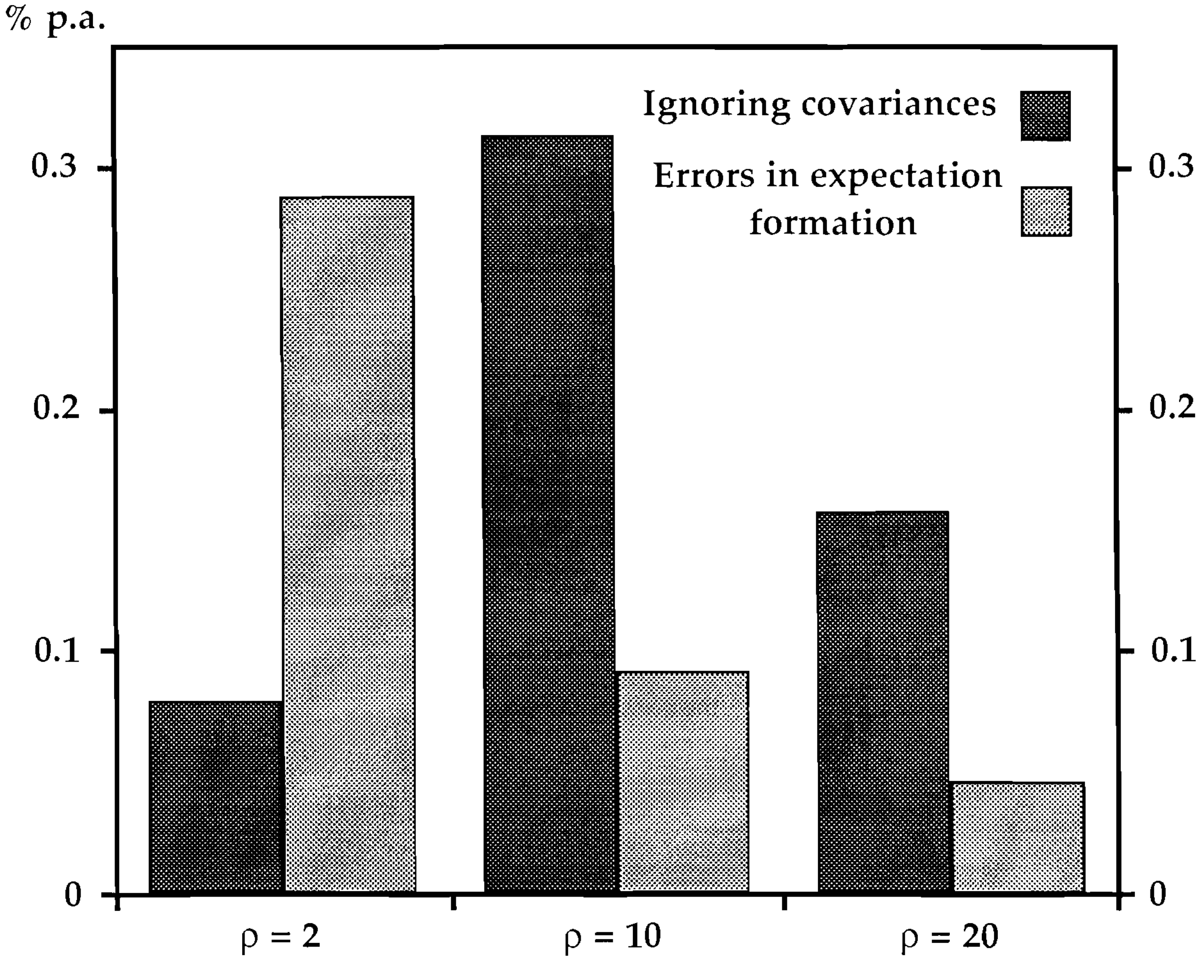
Of course, to estimate the average expected utility derived from these two near-rational strategies, we use the "true" model (which involves using the complete matrix Ω from Table 1 along with equation (2) for exchange rate expectations). Again, the cost associated with a sub-optimal strategy arises from holding inappropriate portfolio shares. Figure 2 shows the costs of these two near-rational strategies.

As in the text, the size of the expected utility costs are small. Assuming $\rho = 2$, the average cost of ignoring the expected covariance between returns is less than 0.1%p.a., while the penalty for making errors in the calculation of expected exchange rate changes of around 1%p.a. is about 0.3%p.a. Note, however, that if the expectational errors are substantially larger than 1%p.a. on average, the expected utility costs will be correspondingly larger. Taken at face value, survey data on exchange rate expectations (Frankel and Froot (1987)) suggest that expectational biases are often substantially larger than 1%p.a.

Figure 2

COSTS FOR NEAR RATIONAL INVESTORS

(All shares are non-negative and foreign shares sum to no more than 100%)



ρ = Coefficient of relative risk aversion

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